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Introduction to games of chance

GambleAware

NSW GOVERNMENT

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Activity introduction

Quick summary

Most of us have played games since we were children. Our first experiences were likely something completely dependent on luck, such as Snakes and Ladders. As we grew and matured, so did our games. They required either physical dexterity, like in Hungry Hippos, or intelligent decision-making, like in Connect Four.

It is inevitable that we eventually decide to make the games more ‘interesting’. We award a prize to the winner. Perhaps they get first dibs on the television, or don’t have to wash the dishes. In a game of pure skill, such as chess, this is a smart decision. Provided, of course, that you are the more skillful player. But what about games of pure luck? Or games that combine luck and skill, such as poker?

In this lesson, students will explore basic probability via games of chance, and will investigate how the gambling industry applies this knowledge to create profit.

Learning intentions

Students will:

- understand how to predict outcomes of games of chance
- understand the concept of a fair game
- understand how these predictions are used to make money in gambling.

Syllabus outcomes

- **MAO-WM-01** develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly
- **MA4-PRO-C-01** solves problems involving the probabilities of simple chance experiments
- **MA4-FRC-C-01** represents and operates with fractions, decimals and percentages to solve problems.

The identified Life Skills outcomes that relate to this activity are **MALS-LAN-01** recognises language that represents number, **MALS-LAN-02** responds to and uses language that represents number, **MALS-FRC-01** demonstrates knowledge of fractions in everyday contexts, and **MALS-PRO-01** applies chance and probability to everyday events.

Capabilities and priorities

Numeracy
Critical and creative thinking
Ethical understanding

Topic
Gambling probability

Unit of work
Mathematics Stage 4

Time required
55 minutes

Level of teacher scaffolding
High - Students will require strong scaffolding through the explicit instruction on calculating probabilities, but will be able to perform the tasks independently.

Resources required

- Dice – one per pair
- Tokens – ten per student
- Pen or pencil for each student
- Whiteboard
- Writing paper or book

Keywords
Gambling, betting, sports, casino, money, wellbeing, gaming.

Teacher worksheet

Teacher preparation

Gambling can be a high-risk activity and is a priority concern for young people. Therefore, before conducting the lesson on gambling, it is recommended that teachers read the Facilitator Pack. The pack provides teachers and parents with essential information about gambling harm amongst young people and clarifies the nature of gambling-related behaviours and how to approach sensitive topics.

Learning intentions

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- understand the concept of a fair game
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Success criteria

Students can:

- calculate the probability of simple events
- use calculations to determine if a game is fair or not.

Teaching sequence

10 minutes - Part A: What is a game of chance?

20 minutes - Part B: What is a fair game?

20 minutes - Part C: Exactly how unfair is it?

5 minutes - Reflection

Part A:
What is a game of chance?

Work through this resource material in the following sequence:

Step 1

Explain to your class that games exist on a spectrum: Some are pure luck, some are pure skill, and most are somewhere in between.

Step 2

On the left side of your whiteboard, write the word ‘Luck’. Write the word ‘Skill’ on the right. Ask your class for games that they feel are based entirely on either luck or skill. Write each game’s name under the appropriate heading.

You may like to use the following examples for clarification:

Snakes and Ladders is a game of pure luck because there are no decisions to be made. You roll a die and move your piece closer to the final space. A fortuitous ladder or ill-fated snake can change your position, but everything is determined by the roll of the die. If you played one thousand games with a friend, you would likely win around five hundred each. There is nothing you can do and no way you can be better than any other player. Success is totally reliant on the roll of the dice.

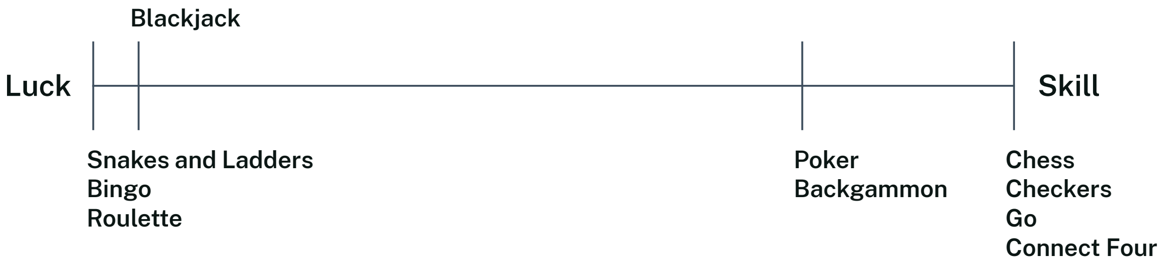
Chess, on the other hand, is a game of pure skill. Both players begin in an equal position, with the same number of pieces in the same positions. The winner will almost always be the more skillful player. You can get better at this game through practice and increased talent, and there are no random elements that can negatively impact that skill level.

Note: Some students may point out that the white player in chess has a slight advantage due to the ability to make the first move. While this is true, that advantage will not help much when there is a noticeable difference in ability between players.

Step 3

Draw a horizontal line across a whiteboard, connecting the words 'Luck' and 'Skill'.
Move the names of games along this continuum.

For example:



You might also include examples such as Hungry Hippos, Twister, Uno, Kahoot, Monopoly, Cluedo or even video games such as Among Us or Solitaire.

Give the class a chance to disagree with any of the placements, and to provide reasons. For example: Some may feel that poker is pure luck, however while the cards drawn involve chance, there are many decisions to be made that depend on a great deal of perception and calculation. The same hand might be worth folding or raising depending entirely on how much money is in the pot, how many players are left in the hand, and how those players have conducted themselves so far.

Step 4

Now ask your class where they feel different sports belong on the spectrum. Where exactly should rugby be placed? It's definitely based on skill, but so many other factors can affect it. Where are they playing? Are any of the players sick or injured? What are the weather conditions? What is at stake, ie: is it a massive game that would get one team into the finals, or a big rivalry?

What about horse racing?

Add a few different sports to the continuum.

Part B:
What is a fair game?

Step 1

Give each student ten tokens, then divide the class into pairs. Give each pair a six-sided die. Explain that they will be playing a simple game of chance and recording the results.

The rules of the game are as follows:

- One player chooses ‘odds’ and the other ‘evens’.
- Draws a two-column table, with ‘Odd’ and ‘Even’ as the two headings. Write the number ‘10’ in each column, below the heading. This represents how many tokens each player has.
- Each player puts a single token into the centre of the play area. This is the cost to play a round (otherwise known as an ‘ante’).
- One player rolls the die. If it comes up 1, 3, or 5, the ‘odds’ player takes both tokens from the middle and adds it to their pile. If the die comes up 2, 4, or 6, the ‘evens’ player takes the tokens.
- Each player now writes down how many tokens they have under the appropriate heading.
- Repeat steps 3-5 until one player runs out of tokens, or five minutes is up.

Step 2

Have a class discussion about the results of the games.

- Did any groups stop playing before the time ran out?
- Did any groups finish the game with ten tokens per player?
- Was this a fair game? Why or why not?

Step 3

The last question is the most important. Mathematically, this is a fair game because both players are equal, and none have an advantage.

Ask students, how do we know this? How can we prove that each player of this game has the same chance of winning? And what are your chances of winning on any given roll of the dice?

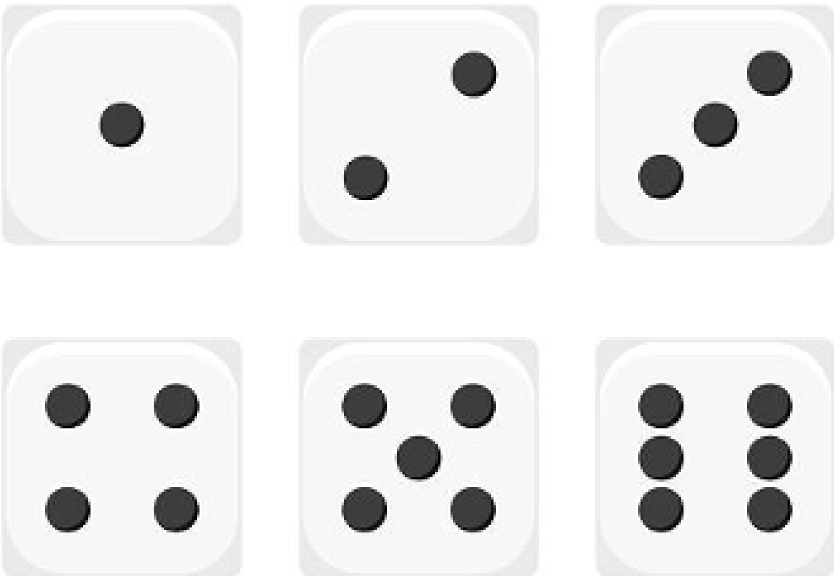
Step 4

We can prove this by calculating the expected value of each round. Given that all outcomes are equally likely when rolling a die, the formula is:

$$P\ (A\ particular\ outcome) = \frac{Number\ of\ ways\ that\ outcome\ can\ occur}{Total\ number\ of\ outcomes}$$

Write this formula on your whiteboard, and show your class how to use it to calculate the probability of rolling a number.

There are six possible outcomes when rolling a fair, six-sided die:



Let's take the role of the 'odds' player. The outcomes they are interested in are 1, 3, and 5. So in this example there are three ways their preferred outcome can occur, and six outcomes in total.

Therefore:

$$P\ (rolling\ an\ odd\ number) = \frac{3}{6} = \frac{1}{2}$$

We use the shorthand P(odd) to represent the probability of rolling an odd number.

So the probability of rolling an odd number is ½, which is the same as 50%. In other words, it should happen half of the time. This is also called an even or 50/50 chance.

Step 5

Explain to your class that when calculating the probability of an event, the answer will always be between 0 (impossible) and 1 (certain). This equates to 0% and 100%. When adding together the probability of all possible outcomes the answer will always be 1, since the probability of every possibility happening is 100%.

In this example we can calculate that the probability of rolling an even number must also be 0.5, because odd and even are complementary events as they cover all the events in the sample space or put another way, when rolling a dice, there are only two possibilities: odd or even

$$P\,(\text{odd}) + P\,(\text{even}) = 1$$

$$\tfrac{1}{2} + P\,(\text{even}) = 1$$

$$P\,(\text{even}) = \tfrac{1}{2}$$

Step 6

Now that we know the probabilities, we can work out how much each player is expected to win or lose every time they play a round. If they lose a round, they lose their original token. If they win a round, they get their original token back, plus their opponent's. In other words, winning or losing a round will either win them, or cost them, a single token.

The expected win/loss per round is found by multiplying the probability of each outcome by the amount you will win or lose, given that outcome. These values are added together for a final result:

Outcome	Probability of outcome	Number of tokens gained or lost	Product
1, 3, 5	½	1	½ × 1 = ½
2, 4, 6	½	-1	½ × -1 = -½
		Expected win/loss (sum of products):	½ + (-½) = 0

The sum of zero means that the expected win/loss is a net gain of zero tokens. Of course this is impossible for a single round, as the only outcomes are winning a token or losing a token, not zero.

However, the expected win/loss is an **average** result when applied to **sufficiently many rounds**. If you played one hundred rounds for example, you should expect to gain a token 50% of the time and lose a token 50% of the time, meaning an average result of 100 x 0 = 0 tokens gained overall.

In other words, this is a fair game.

Part C:
Exactly how unfair is it?

Step 1

Places like casinos wouldn't make much money if their games were fair. A casino (also known as the 'house') must have an advantage on each and every bet made. That means that their expected win/loss per bet must be greater than zero, unlike the game above, so that they can reasonably expect to make a profit over time.

Let's change the rules of our game to reflect this. Instead of having an 'odds' and 'evens' player, we will have a 'player' and the 'house'. The player wins on a 3 or a 5, loses on 2, 4, or 6, and gets their tokens back on a 1.

With your class, calculate the new expected win/loss, from the point of view of the player.

$$\textit{Pr (rolling a 1)} = \frac{1}{6}$$

$$\textit{Pr (rolling a 3 or 5)} = \frac{2}{6} = \frac{1}{3}$$

$$\textit{Pr (rolling an even number)} = \frac{3}{6} = \frac{1}{2}$$

Note that $\frac{1}{6} + \frac{1}{3} + \frac{1}{2} = 1$.

Outcome	Probability of outcome	Number of tokens gained	Product
1	$\frac{1}{6}$	0	0
3, 5	$\frac{1}{3}$	1	$\frac{1}{3}$
2, 4, 6	$\frac{1}{2}$	-1	$-\frac{1}{2}$
		Expected value (sum of products):	$-\frac{1}{6}$

Part C: Exactly how unfair is it?

Over the course of multiple rounds the player can expect to lose one-sixth of a token per round. But since we can't split tokens, looking at it another way, the player can expect to lose one token every six rounds. This game is stacked against the player, and is therefore not fair.

And who is getting this token? The house, of course. It should be easy to look across the class at the end of five minutes and see that each student playing the house has more tokens than the students who are the player.

As an activity, ask your class what could be done to make it fair. Answers might include having a 6 also result in a return of tokens, or making the player win more tokens on a 3 or a 5.

Return to your spectrum of luck and skill based games. Given what you've learned about probability, what does this mean for gambling?

Explain to students that gambling is not like snakes and ladders. The game operators are not hoping you will just have worse luck than them, encountering more snakes while they encounter more ladders.

Gambling is also not like chess where, if you were a grandmaster, you could out-gamble the game operators every time.

Instead, gambling is most like the altered dice game: almost fair, but with a very small chance the players lose and that the game operator makes a profit. Over time, and millions of dollars worth of bets, the game operators make a big profit from this small chance.

Even sports are most like this game. While they're very skill based, there is the smallest chance - due to weather, injuries, or just a freak run of form - that the result will not go as expected, and this is where the game operators profit.

Reflection

Ask students to write a short paragraph regarding unfair games.

- Are they okay if the players know the odds?
- Are they taking advantage of people who aren't aware of the maths?
- Alternatively, who are these systems most advantageous for? Is there any way you could take advantage of the system above?
- Do you think learning this will affect your decision to play certain games in the future?

Teacher reflection

Take this opportunity to reflect on your own teaching:

What did you learn about your teaching today?

What worked well?

What didn't work so well?

What would you share?

Where to next?

How are you going to get there?