4

Prediction vs. reality

February 2024

Activity introduction

Quick summary

We have an instinctive understanding that events should match predictions. Coin tosses should come up half heads, half tails. Six rolls of a die should guarantee each number once. In practice however, we don’t tend to see this.

This lesson will attempt to answer the questions: why doesn’t reality line up with maths, and what will it take to make that happen?

This lesson is designed to teach your students that many people struggle to truly grasp what randomness looks like. Your students will toss coins and record their observations. The trick is that half of your class will fake their results, and you will be able to tell who! They will then explore why this is the case with regards to the Law of Large Numbers.

Learning intentions

Students will:

- understand that mathematical predictions are more accurate over a long period.

Syllabus outcomes

- **MAO-WM-01** develops understanding and fluency in mathematics through exploring concepts, choosing and applying mathematical techniques
- **MA4-PRO-C-01** solves problems involving the probabilities of simple chance experiments
- **MA4-FRC-C-01** represents and operates with fractions, decimals and percentages to solve problems.

The identified Life Skills outcomes that relate to this activity are **MALS-LAN-01** recognises language that represents number, **MALS-LAN-02** responds to and uses language that represents number, **MALS-FRC-01** demonstrates knowledge of fractions in everyday contexts, **MALS-DEP-01** demonstrates knowledge of decimals and percentages in everyday contexts, **MALS-PRO-01** applies chance and probability to everyday events, **MALS-FIN-01** demonstrates knowledge of money in everyday contexts, and **MALS-FIN-02** plans and manages personal finances.

Capabilities and priorities

Numeracy
Critical and creative thinking
Ethical understanding

Topic
Gambling probability

Unit of work
Mathematics Stage 4

Time required
45 minutes

Level of teacher scaffolding
High - students will require strong scaffolding through the explicit instruction on calculating probabilities, but will be able to perform the tasks independently.

Resources required

- Appendix A – Coin toss summary sheet – one per student
- Calculators – one per student
- Coin – one per student
- Student workbooks – one per student
- Whiteboard

Keywords
Gambling, betting, sports, casino, money, wellbeing, gaming.

Teacher worksheet

Teacher preparation

Gambling can be a high-risk activity and is a priority concern for young people. Therefore, before conducting the lesson on gambling, it is recommended that teachers read the Facilitator Pack. The pack provides teachers and parents with essential information about gambling harm amongst young people and clarifies the nature of gambling-related behaviours and how to approach sensitive topics.

Learning intentions

Students will:

- understand that mathematical predictions are more accurate over a long period.

Success criteria

Students can:

- use a model of a series of coin tosses to demonstrate that random events, in reality, do not follow a completely random schedule.

Teacher content information

In a perfect world (perfectly boring) life would be completely predictable. Coin tosses would alternate heads, tails, heads, tails... without deviation. Dice would roll their numbers in order until they reached the end, then would start back again at 1. Reality, however, has other ideas.

Predictions are just that. Predictions. They don't tell the future, rather, they tell us how likely events are. In fact, it's even possible to calculate how likely it is that our predictions are accurate (although that's a whole other matter ...).

Teaching sequence

30 minutes - Part A: Why isn't life more mathematical?

10 minutes - Part B: The law of large numbers

5 minutes - Reflection

Part A: Why isn't life more mathematical?

Work through this resource material in the following sequence:

Step 1

Explain to students that this activity is designed to teach them that many people struggle to truly grasp what randomness looks like.

Students will toss coins and record their observations. However, half of the class will fake their results.

Tell students that you are confident you will be able to tell who faked their results and who actually recorded them!

Step 2

Hand out the coin toss summary sheets (Appendix A).

Step 3

Each student secretly determines if they are going to toss a coin or fake their results. They do this by tossing a coin before starting and looking at the outcome:

- heads - flip for real
- tails - fake it.

Step 4

The students who toss the coin for 'real' are to complete 200 tosses and record the results on the summary sheet.

The students who are faking also complete 200 tosses, but ignore the results and randomly write down a sequence of heads and tails.

Ensure students are recording their results from left to right on the table, using a "H" to represent heads and a "T" to represent tails.

Note: You may wish to do this activity over two days. If you collect the result sheets at the end of the first lesson you could go over them after class. You could also scan the sheets in order to display them to the class later. If time is an issue you could have them record only 100 tosses.

You could also set up a Google sheet and have students record their results electronically, to make the display of the results easier.

Step 5

After your class has recorded the results, collect the recording sheets.

Tell your class you will now attempt to determine whose were real and whose were fake.

Step 6

Go through four or five sheets. Give the students an opportunity to see if they can spot the differences.

Announce whether you think the results are real or fake.

Important: The secret is that the real results will have clusters of consecutive results: six, seven, eight, or even nine heads or tails in a row. The fake results will usually have runs of no more than three.

Ask students to confirm whether they were faking or really recording the results of the tosses.

Step 7

Explain to students that real randomness doesn't look as random as we assume. We tend to think that coin tosses will alternate more than they actually do. The students doing the imaginary tosses are likely to think that four or more heads or tails in a row will look suspicious, and will therefore avoid them in their results, whereas the opposite is actually true: randomness is not evenly spread, but comes in bursts.

Step 8

Display this table. It shows the likelihood of getting a particular number of heads or tails in a row out of 100 or 200 tosses:

Number of heads or tails in a row	Probability - 100 tosses	Probability - 200 tosses
6	80%	97%
7	55%	80%
8	31%	54%
9	17%	32%

As you can see you are almost guaranteed to get a run of 6 heads or tails after 200 tosses.

The reason for this randomness is that these events are *independent*. While independent events are beyond Stage 4 Maths, it is enough to explain to your class that it means the coin tosses don't affect each other. Another way of describing it is by saying the coins have no memory. They cannot remember what the last result was, so each toss will be 'fresh'. The coin doesn't take into account what the previous ten results have been before 'deciding' whether it will land on heads or tails.

Step 9

Ask the 'fakers' in the class how many of them went through their results to make sure they had exactly 50 heads and 50 tails.

Step 10

Ask students how likely a result they think it is. They may be surprised to learn that there is only an 8% chance of getting a perfect 50/50 split. However, if you compare it to the chances of getting every possible number of heads from 0 to 100 (ie: 3 heads, 6 heads, 72 heads), 8% is still the most likely result.

When people make bets on games like roulette or sic bo, they often make the mistake of thinking that if a certain number hasn't appeared in a while it must be more likely to come up in the future. This is called the *gambler's fallacy*. In reality, dice, balls, and coins have no memory. Each roll, toss, or spin has zero impact on the next.

Part B: The law of large numbers

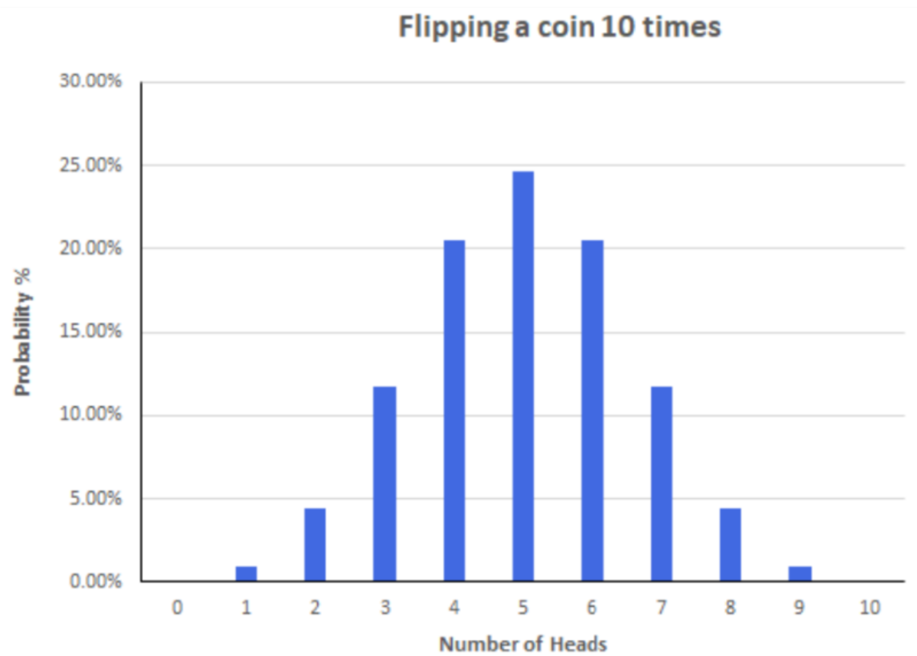
Step 1

Watch: Law of Large Numbers - Explained and Visualized

Step 2

The law of large numbers states that over time, events will more closely match their theoretical probability.

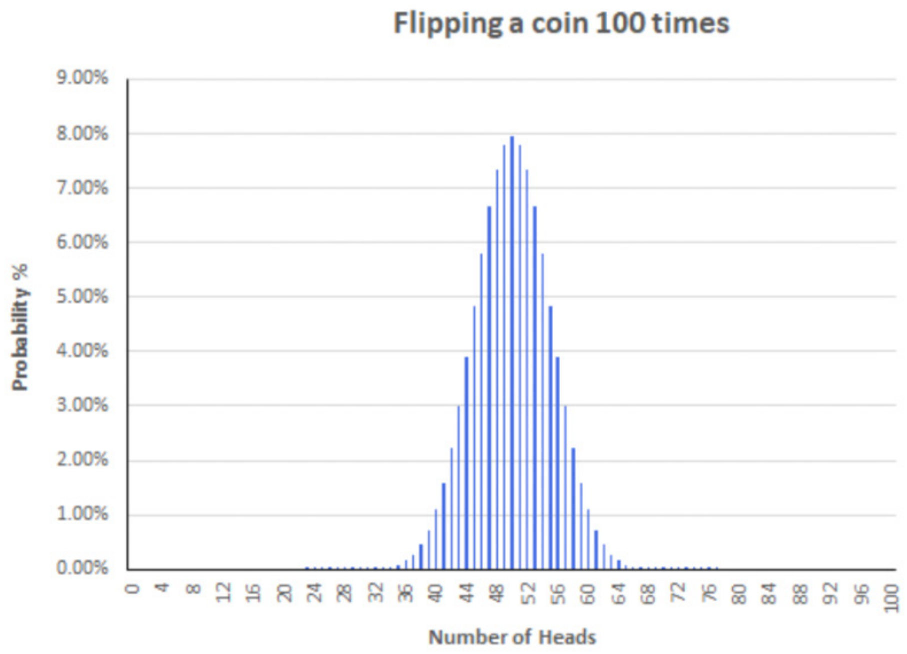
Show the following graph of the probability of getting a certain number of heads after ten tosses.



As we can see, getting five heads out of ten is the most likely outcome (at a smidge under 25%). The probability of getting ten tails is significantly worse, coming in at around 0.1% (or 1 in 1,024). That's not great, but if every student in a school of 1,000 tossed a coin ten times, there's around a 62% chance that it will happen at least once.

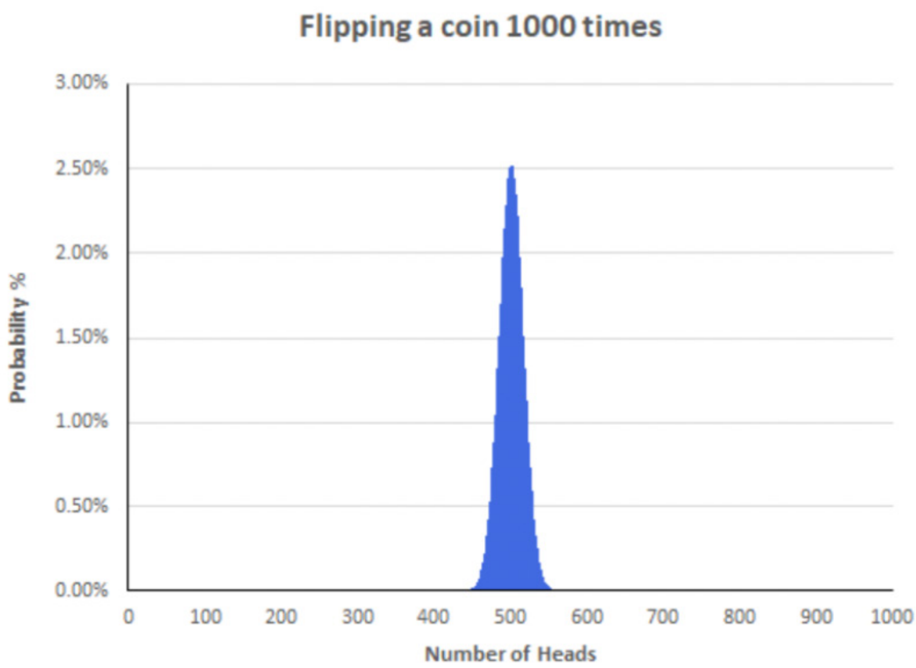
Step 3

Now look at this graph of 100 tosses:



We can see that the probabilities are grouped together much more closely. This is because the more coins we toss in a single experiment, the closer our result will be to the expected outcome: 50% heads and 50% tails.

1,000 tosses and things tighten up even further:



Step 4

When tossing ten coins (or one coin ten times), the chance of getting at least nine heads (90% heads) is just over 1%. Pretty unlikely.

Ask your class what they think the chance of getting at least 90 heads out of 100 tosses is.

They may be surprised to hear that the answer is (1.53×10^{-15})%, or a 1 in 65,289,276,740,295,837 chance (sixty five quadrillion, two hundred and eighty nine trillion, two hundred and seventy six billion, seven hundred and forty million, two hundred and ninety five thousand, eight hundred and thirty seven)!

The chance of getting 100/100 heads is 1 in 1.2 nonillion. That isn't going to happen.

Step 5

Ask your class what this has to do with gambling.

Explain that casinos operate on the assumption that there will be many, many people playing at all times. This means millions of probabilistic events occurring every day: dices being rolled, cards being dealt, wheels being spun…

As far as the Law of Large Numbers is concerned, these are very large numbers indeed! It means that a gambling establishment can predict very accurately how much money they should be making at every table.

In fact, computers are used to count the money being won and lost at each table and will alert casino management if the amount being lost is higher than expected. This is when staff will pay a little more attention to that table to see if anybody might be cheating. That's how reliable mathematical predictions are for large numbers of events. They can be used to catch cheaters!

Reflection

Give your class five minutes to write a paragraph about what they learned today, and why they think it is important. Can they think of any areas in their life where having this understanding of probability would be beneficial?

Teacher reflection

Take this opportunity to reflect on your own teaching:

What did you learn about your teaching today?

What worked well?

What didn't work so well?

What would you share?

Where to next?

How are you going to get there?

Appendix A: Coin toss summary sheet

Toss a coin.

Heads: Toss 200 coins and record the results below, working from left to right.

Tails: Make up your own results, and record them in the table below, working from left to right.

Name									