6

Counter-intuitive probability

GambleAware

NSW GOVERNMENT

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Activity introduction

Quick summary

If you are a human, chances are you are not great at probability. It's not your fault, though. Psychologically speaking we have a lot of things going on in our heads that interfere with it. It's the main reason the gambler's fallacy is so commonplace.

In fact, our inherent difficulties with probability is what allows places like casinos to make so much money. Things that seem obvious can be completely incorrect, and thus lead us into making poor choices.

This lesson is designed to demonstrate the ways in which random chance can be counter-intuitive.

Learning intentions

Students will:

- understand how assumptions made in probability can be risky
- be aware that we often need to perform precise calculations to get an answer.

Syllabus outcomes

- **MAO-WM-01** develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly
- **MA5-PRO-C-01** solves problems involving probabilities in multistage chance experiments and simulations
- **MA5-PRO-P-01** solves problems involving Venn diagrams, 2-way tables and conditional probability.

The identified Life Skills outcomes that relates to this activity is **MALS-PRO-01** applies chance and probability to everyday events.

Capabilities and priorities

Numeracy
Critical and creative thinking
Ethical understanding

Topic
Gambling probability

Unit of work
Mathematics Stage 5

Time required
45 minutes

Level of teacher scaffolding
High - students will require strong scaffolding through the explicit instruction on calculating probabilities, but will be able to perform the tasks independently.

Resources required

- Calculators – one per student
- Student workbooks

Keywords
Gambling, betting, sports, casino, money, wellbeing, gaming.

Teacher worksheet

Teacher preparation

Gambling can be a high-risk activity and is a priority concern for young people. Therefore, before conducting the lesson on gambling, it is recommended that teachers read the Facilitator pack. The pack provides teachers and parents with essential information about gambling harm amongst young people and clarifies the nature of gambling-related behaviours and how to approach sensitive topics.

Learning intentions

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- be aware that we often need to perform precise calculations to get an answer.

Success criteria

Students can:

- perform calculations based on probability word-problems.

Teaching sequence

5 minutes - Part A: The librarian and the farmer

20 minutes - Part B: It's a girl!

15 minutes - Part C: Pillow-problem number five

5 minutes - Reflection

Part A:
The librarian and the farmer

Work through this resource material in the following sequence:

Step 1

Give your students the following problem:

> *'A person chosen at random is quiet, introverted, focused on details, and helpful. Are they more likely to be a librarian or a farmer?'*

Step 2

Give students time to discuss the answer amongst themselves. Then discuss as a class, asking some students to volunteer their reasoning.

Step 3

The obvious answer is 'librarian'. Unfortunately, that is incorrect.

As counter-intuitive as it sounds, the reason is very simple. There are approximately 2.6 million libraries across the world, and at least 570 million farms.

Step 4

In some ways your students might feel cheated by this question, but explain that they're in a maths class, not a social science class, and that you're going to be looking at and making decisions based on probabilities - just like you should when you're gambling.

Part B:
It’s a girl!

Step 1

Tell your class that you have a friend with three children. Ask them:

1. What is the probability that they are all girls?
2. What is the probability that they are all girls, given that the eldest is a girl?
3. What is the probability that they are all girls, given that at least one is a girl?

Step 2

Give students time to come up with answers to these questions, either individually or in small groups. Ask them to show the maths that has led them to these answers.

Step 3

Discuss as a class, asking some students to volunteer their reasoning.

How many students believe that all three questions have the same answer: $\frac{1}{8}$?
They may be surprised to hear that it is only true for the **first** one.

Let’s scaffold students through each of these problems.

Step 4

1. What is the probability that they are all girls?

Tell students to assume that the probability of a child being a girl is $\frac{1}{2}$ (which it isn’t exactly, but close enough).

Independently, students to find the probability of

The first child being a girl: $\dfrac{1}{2}$

The second child being a girl: $\dfrac{1}{2}$

The third child being a girl: $\dfrac{1}{2}$

Part B: It's a girl!

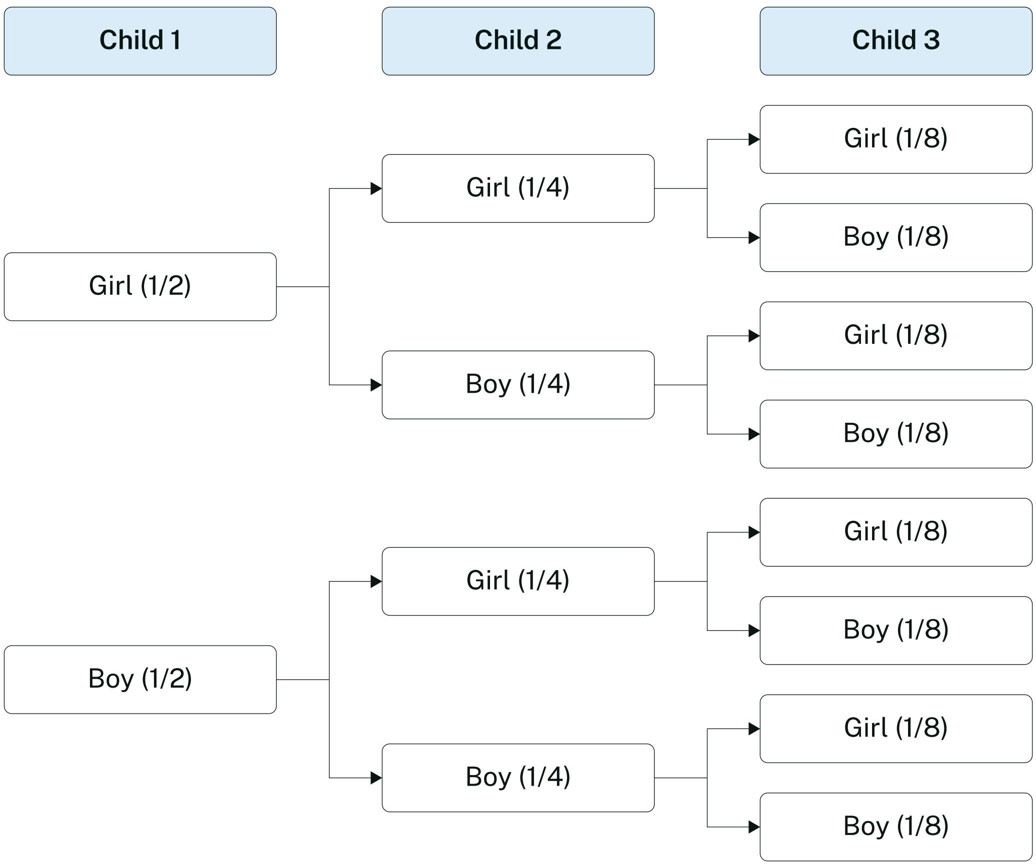
Step 5

Now ask students to find the chance of three children in a row being girls:

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

Step 6

Independently, students construct a tree diagram of all the possible outcomes of your friend's children's genders.



Step 7

Next, students find the probability of each of these outcomes.

Remind students that there is always a 50% chance the next child being a boy or a girl, so the probability halves for each new child.

Each possible outcome of three kids {GGG,GGB,GBG,BGG,GBB,BGB,BBG, BBB} are all equally likely, so the chance of GGG is ⅛.

Step 8

2. What is the probability that they are all girls given that the eldest is a girl?

In this case we are just calculating the chance of the first two children being girls, which is:

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

We could also look at the sample space and see that only four of them have a girl as the third child: {GGG, GBG, BGG, BBG}. Of these, only one is GGG, therefore the probability is ¼.

Step 9

3. What is the probability that they are all girls given that at least one is a girl?

Direct students to look at the sample space again.

Ask students to find all the outcomes where there is at least one girl, and eliminate any in which there aren’t any girls. They will only have to disregard BBB, the only option that doesn’t meet our requirements. There are now seven possibilities to consider: {GGG,GGB,GBG,BGG,GBB,BGB,BBG}.

Of these, only one is GGG, therefore the probability is 1⁄7.

Note: This can also be shown using the conditional probability formula:

$$P(\textit{Three girls} \mid \textit{At least one girl}) = \frac{P(\textit{Three girls})}{P(\textit{At least one girl})}$$

$$P(\textit{Three girls}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$\begin{aligned} P(\textit{At least one girl}) &= 1 - P(\textit{No girls}) \\ &= 1 - P(\textit{Three boys}) \\ &= 1 - \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) \\ &= 1 - \frac{1}{8} = \frac{7}{8} \end{aligned}$$

Therefore: $P(\textit{Three girls} \mid \textit{At least one girl}) = \dfrac{\frac{1}{8}}{\frac{7}{8}} = \dfrac{1}{7}$

Part C: Pillow-problem number five

Step 1

Ask your class what they know about Lewis Carroll, author of the Alice in Wonderland book. They may not be aware that he was a keen mathematician, and wrote a book called Pillow-Problems, that contains 72 mathematical puzzles.

Step 2

Go through problem #5 with your class:

> *'A bag contains a counter, known to be either white or black. A white counter is put in, the bag is shaken, and a counter is drawn out, which proves to be white. What is now the chance of drawing a white counter?'*

In other words, we have a bag with a single counter or token in it, with a 50% chance of being white, and a 50% chance of being black. A second counter is added, and we know that it is white. The bag is shaken to mix the counters around. One counter is then removed. It is white. What is the probability of the remaining counter also being white?

Step 3

Independently, students work on solving this problem. The obvious answer is a probability of ½, but intuitive students will have picked up by now that the answer can't be that simple. This answer is due to the misleading observation that nothing has changed. A white counter was put in and a white counter was withdrawn. This makes it appear as if the remaining counter must be the original one, with a ½ probability of being white.

Step 4

Share the following table with students.

Colour of original counter	Colour of introduced counter	Counter which was drawn out (original/ introduced)	Chance of this occurring

Independently, students find all the possible outcomes of this scenario. Remind students to leave the last column until they are certain they have found all possible outcomes.

The answer has been included here for you:

Colour of original counter	Colour of introduced counter	Counter which was drawn out (original/introduced)	Chance of this occurring
Black	White	Introduced	⅓
White	White	Introduced	⅓
White	White	Original	⅓

Step 5

Re-read Carrol's problem to students, and ask:

- What is the chance the original counter is white, given that a white counter has been taken out of the bag?

Two of the outcomes we have found meet these conditions. The three cases above are all equally likely.

Therefore the correct probability of the original counter being white after a white counter is withdrawn is ⅔.

Reflection

Ask students whether they gave the obvious, but incorrect, answer to any of these questions? If so, how did that make them feel?

Did they still feel like they were right, even though the maths showed clearly how they weren't?

Did they continue to look for an answer which suited their intuition?

Did they feel cheated at all, confident they had the right answer and losing seemingly despite this?

Remind students that gamblers are more likely to experience gambling harm when they feel like their 'knowledge' about games of chance gives them an advantage, and that this lesson demonstrated how probability is, in reality, often the opposite of what we are sure we know.

Independently, students write a short paragraph reflecting on this prompt:

Probability can often feel counter-intuitive - going against what we think or expect.

How might this impact our approach to gambling?

Prompt student thinking further by asking: what are some things gamblers are reasonably sure they know (a sporting team being better than another team, what card is going to come up next, the odds of a dice roll). How might chance or other variables impact this knowledge and lead to an outcome that feels counter-intuitive?

Teacher reflection

Take this opportunity to reflect on your own teaching:

What did you learn about your teaching today?

What worked well?

What didn't work so well?

What would you share?

Where to next?

How are you going to get there?