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Should you swap?

NSW GOVERNMENT

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Activity introduction

Quick summary

This classic probability problem, commonly known as the Monty Hall problem after the host of the original game show, has divided gamblers - and even some mathematicians! - for decades. The premise is simple: having chosen what you think is the winning door with the money behind it, should you swap to another door when Monty offers you the opportunity?

Students will first use probability language to define the problem. Students will then construct a mathematical model and run a number of trials before collating their data to determine whether the probabilities match the reality of experiencing the situation. Finally, students will consider what lessons from the Monty Hall problem can be generalised to other gambling behaviours, including trusting the mathematics behind gambling, and how other 'human factors' can influence gamblers towards riskier behaviours and decisions around their gambling.

Learning intentions

Students will:

- understand that the probability of an event can be determined theoretically, from long-term statistics or from modelling with a sufficient number of trials.

21st-century skills

Critical thinking

Flexibility

Initiative

Problem solving

Teamwork

Syllabus outcomes

Mathematics Standard (Year 11)

- **MS11-6** makes predictions about everyday situations based on simple mathematical models
- **MS11-7** develops and carries out simple statistical processes to answer questions posed.

Mathematics Extension 1 (Year 11)

- **ME11-6** uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts.

Mathematics Life Skills (Years 11 and 12)

- MLS-S2 probability.

General capabilities

Numeracy

Critical and creative thinking

Ethical understanding

Stage 6 Mathematics Syllabus Statements

Students develop awareness of the applicability of algebra in their approach to everyday life. Students analyse different financial situations, to calculate the best options for given circumstances, and solve financial problems. They develop the ability to make informed financial decisions, to be aware of the consequences of such decisions, and to manage personal financial resources effectively. Students develop an ability to justify various types of financial decisions which will affect their life now and into the future.

Knowledge of statistical analysis enables the careful interpretation of situations and raises awareness of contributing factors. Study of statistics is important in developing students' understanding of the contribution that statistical thinking makes to decision-making in society and in the professional and personal lives of individuals.

Students model theoretical or real-life situations involving algebra. Students develop knowledge, skills and understanding to manipulate, analyse and solve polynomial equations. Students use algebraic and graphical techniques to describe and solve problems and to predict outcomes with relevance to, for example, commerce.

Students develop an understanding of the language and elements of chance and probability and apply this in real situations.

Activity introduction

Topic
Probability

Unit of work
Mathematics Stage 6

Time required
55 minutes

Level of teacher scaffolding
Medium – students will need to be heavily scaffolded through the explicit teaching of the probabilities of this trial. However, they will require minimal support when running the trial.

Resources required

- Device capable of presenting a video to the class
- Dice – 6 sided – One per student
- Student workbooks

Keywords
Gambling, betting, sports, casino, money, wellbeing, gaming, Monty Hall, probability.

Teacher worksheet

Teacher preparation

Gambling can be a high-risk activity and is a priority concern for young people. Therefore, before conducting the lesson on gambling, it is recommended that teachers and parents read the Facilitator pack. The pack provides teachers and parents with essential information about gambling harm amongst young people and clarifies the nature of gambling-related behaviours and how to approach sensitive topics.

Learning intentions

Students will:

- understand that the probability of an event can be determined theoretically, from long-term statistics or from modelling with a sufficient number of trials.

Success criteria

Students can:

- create a model of an event to verify its theoretical probability is correct
- give an example of where the probability of an event can be determined theoretically, from long-term statistics or from modeling.

Teaching sequence

20 minutes - Part A: Establishing the scenario

10 minutes - Part B: Running a model to simulate the event

15 minutes - Part C: Collating and verifying the results

10 minutes - Part D: Reflection

Part A:
Establishing the scenario

Work through this resource material in the following sequence:

Step 1

Explain to students the following scenario.

> *You are a contestant on a game show. You are given the choice of 3 doors.*
>
> *Behind 2 of the doors is a goat, but behind the other is a major cash prize.*
>
> *You choose Door A.*
>
> *The host checks both the other doors and then opens Door B to show you there is a goat.*
>
> *The host then asks if you would like to stay with your original choice, Door A, or perhaps swap to Door C, the remaining unopened door.*

Step 2

Have the students write the question in their own words.

Before allowing students to form their own opinion, ask them to write down the question in their own words and share with a partner. This is just to make sure they all have the same understanding of the problem. The phrase, *"The host checks both the other doors"* is a very important part of the understanding here. The host knows if the money is behind Door C, that is why he shows that Door B is a goat. Of course, the money could also be behind Door A.

Have several students read the question exactly as they wrote it and seek feedback from the class.

Check they are very specific about who knows what and the steps involved.

It is important that everyone has the correct understanding.

Step 3

Instruct the students as follows:

> *Now you have a couple of minutes thinking time to formulate your answer to the question:*
>
> *"Should you swap doors to Door C? Or should you stick with your original choice of Door A?"*

Give students some time to independently establish their own position.

Students can then discuss their answer with a partner.

Step 4

Invite some students to volunteer their answer and justify their thinking using the language of probability, and discuss as a class.

Students may have their own (strong) opinions and reasoning, however, these generally fall into three categories. Take the time to discuss the three different positions (as well as any other student suggestions):

Position 1: It makes no difference to stay or change.
There are 2 doors remaining so each has a 50/50 chance. There is no advantage to swapping.

Position 2: You should stay with Door A.
Since there are 2 doors remaining each has a 50/50 chance. You may feel the host is trying to influence you into swapping because they know you have selected the correct door.

Position 3: You should change.
With your original choice you had a ⅓ chance of choosing the money (1 money, 2 goats).

Therefore the chance of the money being in one of the other two doors is ⅔ (Door B chance + Door C chance).

By eliminating Door B, which had to be a goat, the host has actually concentrated that chance solely into Door C. Door C now has twice the chance of containing the money as Door A so you should definitely change.

This (swapping doors) is in fact the correct answer, at least mathematically speaking.

Step 5

Now ask students to vote: Are they going to stay with Door A, or switch to Door C?

Step 6

Watch: 21 - Monty Hall - Propensity based theoretical meodel probability - mathematics in the movies

(*Source:* youtube.com/watch?v=iBdjqtR2iK4)

This clip explains the answer reasonably well.

However, students might not be convinced, as the problem invokes strong opinions. You might not even agree with the answer! Many staff and students may want to argue.

Modelling the situation can prove the correctness of the answer given.

Part B: Running a model to simulate the event

Step 1

Now pose the question:

"How could we find out which answer is correct?"

You are looking for an answer that can be interpreted as:

"We should do it and see what happens?"

Explain to students this is called mathematical modelling, and yes, it would be a good idea to do that here.

Step 2

Independently, or in small groups, students will now test the game show.

You may first wish to quickly discuss how many times you'd need to run the model for you to be confident in the result. The answer is the more the better but 50 should be enough trials to see the pattern clearly.

You are going to be combining the results of the whole class in order to be time efficient whilst also gathering a large quantity of data.

Step 3

Students roll a six-sided die to determine what door the money is behind for each trial.

1 or 2: Door A

3, 4, 5, 6: Door C

For the first ten rolls, imagine the contestant on the quiz show has swapped to Door C.

For the next ten rolls, imagine the contestant has stayed with Door A.

Record whether that choice resulted in them winning the money or losing by getting the goat, based on where the die roll determined the money was.

Step 4

Students record their trials in a table such as this:

	Number of trials	No. of wins (Money in this door)	No. of losses (Goat in this door)
Swap (Door C)	10		
Don't swap (Door A)	10		

Part C: Collating and verifying the results

Step 1

Once each student has run 20 trials, collate the results of every student or group on a single table on the board.

For example:

	Number of trials	No. of wins (Money in this door)	No. of losses (Goat in this door)
Swap (Door C)	100	67	33
Don't swap (Door A)	100	39	61

Step 2

As a class, discuss the results.

They should roughly indicate that you win twice as often as you lose (66% wins compared to 33% loses) when you swap to Door C.

This is supported by the mirror trial where you lose twice as often when you decide to stay on Door A.

Step 3

Why might the results not exactly match these percentages? Well, reality does not always match the maths of probability. Also, you have only run a relatively small number of trials. If you were to conduct 1 million, or 1 billion trials, you would find the results to be much closer to a split.

Step 4

Has conducting the trials convinced students? Or do they (and you!) still remain convinced that there is a trick going on here somewhere?

Spend some time discussing this lesson.

Part D: Reflection

When this problem was first proposed, there was huge controversy as to its solution, with even mathematicians divided.

This might come down to something that is common throughout gambling, which is that decisions on bets are often made on more than just the probability involved.

Discuss why this solution might be controversial, and how it explains decisions on other forms of gambling. Prompt student thinking with the following:

- Gamblers might feel that the game show host is trying to influence the contestant into swapping because they have seen behind both doors and know that the contestant will be right if they stay, so is trying to convince them so that the game show doesn't lose money. This is the human factor, and can conflict with what we know as the right choice based on mathematical probability.
- The longer a bet stands, the more a gambler might start to question their decision and be tempted to change or withdraw their bet.
- Conversely, a gambler might think they'd feel terrible if they changed and their original decision proved correct, thereby convincing themselves to 'stick to their guns'.
- Despite the maths, gamblers might feel that an improbable result 'has to' come up eventually. But remember: numbers don't have memories. They just do what probability tells them to do.
- Gambling based on any of these factors, outside of the probability, is therefore illogical.

Differentiated learning

- If students remain unconvinced, change the problem slightly. There are now 100 doors. The student chooses one. The probability of success is just 1 in 100. The chance of it not being there is 99 out of 100. If the host removes 98 of the remaining doors and they are all goats, should you swap now?
- For students who may require additional support it may be necessary to run the trials together. The teacher plays the role of the game show host, rolling the dice to determine which door the money is behind. Choose a different student as the contestant for each trial. Will they swap, or stay? Record the results for the whole class, and see whether it accurately reflects the expected outcome.

Extension

- Instead of giving students the methodology for the trials, challenge students to independently come up with their own trial. Have them justify their approach, and after they have completed their trials, discuss whether their personal results accurately reflect the expected outcome.

Teacher reflection

Take this opportunity to reflect on your own teaching:

What did you learn about your teaching today?

What worked well?

What didn't work so well?

What would you share?

Where to next?

How are you going to get there?