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Easter Show games of chance

GambleAware

February 2024

Activity introduction

Quick summary

Playing games of chance at the Easter Show can be a lot of fun - as long as you understand it is unlikely that you will win in the long run. The operator of these games has an advantage built in to ensure their own profit, just like legalised gambling.

In this lesson, students will calculate the probability of an average person scoring a shot at a basketball game at the Easter Show. They will then use these probabilities to design a payout system which can absorb the losses from an average player winning big, whilst profiting from the average player who scores very poorly, and thereby understand how game operators ‘play the odds’ to remain profitable. They will then generalise this understanding to other forms of gambling, specifically poker machines, to understand how gambling companies profit from gamblers in the long run even whilst paying out big jackpots. Finally, they will reflect on how this informs their approach to gambling.

Learning intentions

Students will:

- understand that the probability of an event can be determined theoretically by a suitable calculation or by collecting long term data
- understand that the designer of games of chance uses these probabilities to set the prizes, thereby making sure that the operator will win in the long run.

21st-century skills

Communicating

Creative thinking

Critical thinking

Entrepreneurship

Ethical behaviour

Flexibility

Problem solving

Syllabus outcomes

Mathematics Standard (Year 11)

- **MS11-7** develops and carries out simple statistical processes to answer questions posed.

Mathematics Standard (Year 12)

- **MS1-12-3** interprets the results of measurements and calculations and makes judgements about their reasonableness
- **MS1-12-4** analyses simple two-dimensional and three-dimensional models to solve practical problems.

Mathematics Life Skills (Years 11 and 12)

- **MLS-S2** probability.

Stage 6 Mathematics Syllabus Statements

Students develop awareness of the applicability of algebra in their approach to everyday life. Students analyse different financial situations, to calculate the best options for given circumstances, and solve financial problems. They develop the ability to make informed financial decisions, to be aware of the consequences of such decisions, and to manage personal financial resources effectively. Students develop an ability to justify various types of financial decisions which will affect their life now and into the future.

Knowledge of statistical analysis enables the careful interpretation of situations and raises awareness of contributing factors. Study of statistics is important in developing students' understanding of the contribution that statistical thinking makes to decision-making in society and in the professional and personal lives of individuals.

Students develop an understanding of the language and elements of chance and probability and apply this in real situations.

Activity introduction

Topic
Probability

Unit of work
Mathematics Stage 6

Time required
55 minutes

Level of teacher scaffolding
Medium – Students will need to be heavily scaffolded through the explicit teaching of the probabilities involved in this scenario. However, they will require minimal support when making their calculations.

Resources required

- Appendix A – Calculating percentages of winners – two per student
- Appendix B – Calculating game operator profit/loss – two per student
- Calculators – some students might require calculators to multiply fractions
- Student workbooks

Keywords
Gambling, betting, sports, casino, money, wellbeing, gaming, probability, sideshow, Easter, jackpot.

Teacher worksheet

Teacher preparation

Gambling can be a high-risk activity and is a priority concern for young people. Therefore, before conducting the lesson on gambling, it is recommended that teachers and parents read the Facilitator pack. The pack provides teachers and parents with essential information about gambling harm amongst young people and clarifies the nature of gambling-related behaviours and how to approach sensitive topics.

Learning intentions

Students will:

- understand that the probability of an event can be determined theoretically by a suitable calculation or by collecting long term data
- understand that the designer of games of chance uses these probabilities to set the prizes, thereby making sure that the operator will win in the long run.

Success criteria

Students can:

- calculate the probability for success in the basketball toss Easter Show game
- discuss the probability of success in the basketball toss Easter Show games using appropriate terms
- make connections between the probability of success in these games and the way the gambling industry designs games and betting schemes to profit in the long run.

Teaching sequence

10 minutes - Part A: Games of chance

30 minutes - Part B: Shooting basketballs

10 minutes - Part C: Poker machines, skill, chance, and payout probabilities

5 minutes - Part D: Reflection

Part A:
Games of chance

Work through this resource material in the following sequence:

Step 1

Begin by asking students what games of chance they have seen at the Easter Show, usually located in the sideshow alley.

Write a list of suggestions on the board. This may include, but is not exclusive to:

- shooting basketballs into rings
- throwing darts at balloons
- knocking down a pyramid of cans with a bean bag/ball
- shooting a pea gun at moving or stationary targets
- raffles
- throwing a hoop over a cylinder
- putting ping pong balls into the mouth of a laughing clown.

Step 2

Ask students how they rate their chances of winning at these games. Put a ranking (out of ten) next to each of the games.

For example, students might feel like their skills at dart throwing will give them an advantage at that game, with a 7/10 chance of winning a prize.

On the other hand, the laughing clowns seem a lot more random and your skills don't play into the result, so the chances of winning a prize are 3/10.

Step 3

Explain to students that, in all the games, success is a possibility.

However, for most of these games, operators are relying on the long term statistics to ensure they make a profit more often than they are rewarding or paying out the players.

In some cases, the games may not be what they seem, with built in advantages for the operator.

For example:

- **Throwing darts at balloons.**
  Often the darts are blunt and the balloons are only partially blown up, so even if you hit directly the dart will likely bounce off and not win you a prize.

- **Shooting a pea gun at moving or stationary targets.**
  The sight on the "gun" can be off centre, making it much harder to aim.

- **Knocking down a pyramid of cans with a bean bag.**
  The lowest level of cans are weighted down with sand, creating a very stable base for the pyramid, which is hard to knock down, even with direct hits.

We are going to look more closely at two of the more "fair" games, shooting basketballs and laughing clowns, and how they are still designed to favour the operator and not the player.

Part B: Shooting basketballs

Step 1

Begin by establishing the game. This game takes many forms and students may suggest different rules, but for the sake of consistency, we're going to run it with the following rules:

- The player buys three shots at a time. It costs $1 for each shot, for a total of $3 per play.
- Shoot each of your three basketballs and count the number of successes (hoops you make).
- Your prize is dependent upon the number of successes from your three shots. The prizes increase in value as the number of successes increases. So, 3/3 hoops made will reward you with the best prize.
- Shooting 3/3 will win you the major and best prize, 2/3 a pretty good prize, and so on, on a sliding scale.

Note: If you accept another version of the game, you will need to adapt the maths that follows accordingly, which is why this is not recommended.

Step 2

We first need to determine all the possible outcomes or results for this game. Ask students to suggest every possible score a player could achieve with their three shots. We will call this outcome, across **all three shots**, "X".

There are four possible outcomes:

- X = 0/3
- X = 1/3
- X = 2/3
- X = 3/3

It is important that everyone has the same understanding about this. X is the number of successes **in 3 throws**, which is different to the outcome of a single throw.

Step 3

Independently, students list all the possibilities for three individual throws.

Encourage students to look for patterns so that they cover all results (there are eight different possible combinations). For example, if you always miss the first shot, what could the other two shots be?

Provisions for learning support

It is helpful for students to think of the three basketballs as different colours.

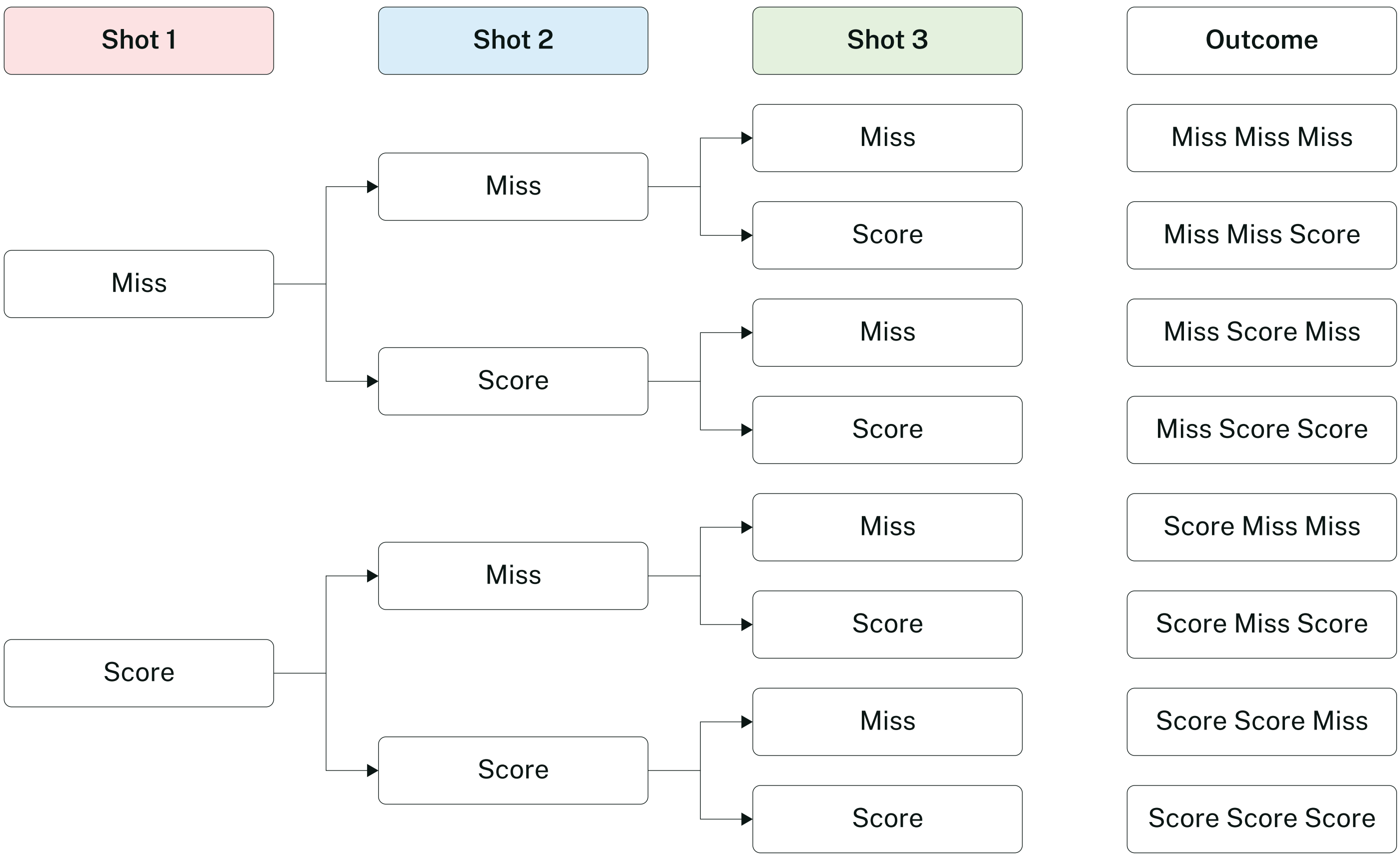
You can share a version of the table on the following page to support students in finding all possible outcomes, and share the first result with them as an example.

Part B: Shooting basketballs

The answers are listed here for you.

X	Shot 1	Shot 2	Shot 3
X = 0	Miss	Miss	Miss
X = 1	Miss	Miss	Score
X = 1	Miss	Score	Miss
X = 2	Miss	Score	Score
X = 1	Score	Miss	Miss
X = 2	Score	Miss	Score
X = 2	Score	Score	Miss
X = 3	Score	Score	Score

You could also use a tree diagram to discover all results.



Important: Is Score Miss Miss the same outcome as Miss Score Miss?

For the purposes of this calculation, we need to know all the possible combinations of success and failure, so yes, they are different, even though they're achieving the same thing.

This is why the coloured balls may help some students: red score, blue miss, green miss will appear different to red miss, blue score, green miss.

Step 4

So there are four different possible 'prize scores' (X = 0, 1, 2, 3) and eight different possible 'score combinations' depending on which of our shots score or miss.

Ask students:

- Do we have the same probability of scoring a 3 as we do of scoring a 1?

Taking away all of the superstar basketballers you might have in your class, another way of asking this question is:

- Who thinks they could score once in three shots? Who thinks they could score on two shots out of three, remembering you can have your miss with any shot? Who thinks they could score on all three shots in a row?

The pressure would definitely be on knowing you had to make three baskets without a miss!

Step 5

Ask students to arrange the outcomes from Step 3 into 'prize scores' by listing the score configurations using the table (available in Appendix A).

The answers have been provided for your convenience.

Number of successes ('scores')	List of different configurations to achieve this number of successes	Calculation of probability of achieving this number of successes	Number of players who achieve this prize score	% of players who achieve this prize score
0	MMM			
1	SMM, MSM, MMS			
2	SSM, SMS, MSS			
3	SSS			

Let's say, on average, a player could make one in every four shots, or 25% of the shots they take.

This means that for any given shot:

$$\frac{1}{4} = \text{Score,} \qquad \frac{3}{4} = \text{Miss.}$$

Step 6

Independently, students calculate the chance of achieving each score configuration based on the chances of scoring or missing with their first, second, and third shot.

Before students begin, let's work through the first configuration together.

$$\text{Miss Miss Miss} = \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$$

This means that out of every 64 players of the basketball game at the Easter Show, on average, 27 miss every single shot.

Step 7

Ask students to complete the rest of the table, converting the number of players who achieve each prize score into a percentage by dividing the numerator by the denominator and multiplying by 100.

The answers have been provided for your convenience.

Number of successes (prize score)	List of different configurations to achieve this number of successes	Calculation of probability of achieving this number of successes	Number of players who achieve this prize score (out of 64 players)	% of players who achieve this prize score
0	MMM	$\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$	27	42%
1	SMM, MSM, MMS	$\frac{1}{4} \times \frac{3}{4} \times \frac{3}{4} \times 3 = \frac{27}{64}$	27	42%
2	SSM, SMS, MSS	$\frac{1}{4} \times \frac{1}{4} \times \frac{3}{4} \times 3 = \frac{9}{64}$	9	14%
3	SSS	$\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$	1	2%
Total:	8		64	100%

The total number of players should equal 64. If they don't, students have made a mistake somewhere.

Important: For the prize scores of 1 and 2, we have multiplied these probabilities by three, since there are three different ways this prize score can be achieved.

Some students may prefer to calculate the probability of achieving each of the eight different combinations, but remind students that:

- They will be the same probability, even though the scores and misses are in three different orders/ combinations.
- They will need to be added together to find the true probability of achieving that score.

Step 8

Now present students with the following table (available in Appendix B).

Prize 'score'	Number of 'winners' of this prize category	Value of sales ($3 per game)	Value of prize	Total cost of prizes	Profit/Loss for game operator
0					
1					
2					
3					
Total:					

We know the number of people who, on average, will score either 0, 1, 2, and 3.

We know each person pays $3 to play the game.

Independently, students can fill out the first two columns of this table.

Step 9

Now ask students to think from the perspective of the game operator. They want to make a profit from people playing their game.

Independently, students set the value of the prizes for each category.

For example, the major prize, for scoring on all three shots, might be a giant teddy bear worth $15.

The prize for missing all three shots (0) might be a plastic whistle worth $1.

Allow students to set any prices they like for each prize 'score', and calculate the total cost of the prizes the operator would expect to spend based on the average number of people the operator expects to win that prize.

Step 10

Finally, ask students to calculate the profit or loss of each prize category by subtracting the cost of the prizes from the 'ticket' sales of people who paid to play the game.

An example of this table has been included on the following page.

Part B: Shooting basketballs

Prize 'score'	Number of 'winners' of this prize category	Value of sales ($3 per game)	Value of prize	Total cost of prizes	Profit/Loss for game operator
0	27	27 × $3 = $81	$1	27 × $1 = $27	$81 - $27 = +$54
1	27	27 × $3 = $81	$2.50	27 × $2.50 = $67.50	$81 - $67.50 = +$13.50
2	9	9 × $3 = $27	$4	9 × $4 = $36	$27 - $36 = -$9
3	1	1 × $3 = $3	$15	1 × $15 = $15	$3 - $15 = -$12
Total:	**64**	**$192.00**	-	**$145.50**	**+$46.50**

Step 11

Let's analyse these results.

- Because the game operator knows most people will score 0 or 1, so long as the prizes cost less than the cost of playing the game ($3), the game operator will make a profit.
- Players who score all 3 shots will rightly expect a big 'jackpot' prize, which has to be more expensive than their cost of entry. Obviously the game operator is going to make a big loss whenever they have to pay out this jackpot.
- However, make a point to students that the game operator is playing the odds. They know that, on average, only 2% (1 in every 64 players) are likely to win big and make all three shots,so this is an acceptable risk.
- We can see that even with paying out big prizes to those players who score on 2 or 3 of their shots, the game operator is still going to make a profit overall for every 64 players who play their game.

Step 12

Ask students, did anyone have any different values for the prizes in each of the prize categories?

Depending on their answers, you might prompt their thinking by discussing:

- Missing all three shots (scoring 0) should not win you a prize. But would this make the game attractive for people to play?
- You could change the values of the prizes to make even more profit. For example: 0 = $0, 1 = $1, 2 = $2, 3 = $5. However, again, does this make the game attractive to play? Players have to be tempted by the risk vs reward of the game. They have to be made to think, if only they can make three shots, they could get a big, big reward.
- In fact, the prize for making 3 shots could be worth $52 (using the other values in the table above), and the game operator, with a strong knowledge of the probabilities involved, could still expect to make a profit (albeit of only $0.50.)

Step 13

Now there are two challenges to consider with this game.

- The game operator can (and does) manipulate the game to ensure even less people score on their shots. For example, they can make the rims of the hoops and the balls smaller than the standard for basketball to throw people off. They can manipulate the perspective so that the hoops look closer than they are. And they can overinflate the balls, making them bouncier and less likely to drop.
- What about the student in your class who, when asked earlier, said they were a gun baller, capable of shooting at more than 25%? Let's call him Mike J. If the game operator saw Mike coming down sideshow alley at the Easter Show, would he suddenly be worried about his system? Well, let's investigate.

Step 14

Distribute another copy of Appendix A and Appendix B to each student.

Step 15

The list of different configurations to achieve this number of successes will remain the same as last time. However, the calculations will use different values.

Let's say in every 3 throws Mike gets 1 hit and 2 misses ($\frac{1}{3}$ = Score, $\frac{2}{3}$ = Miss)

Independently, students calculate the likelihood of Mike scoring 0, 1, 2 or 3.

The results are included for you below:

Number of successes (prize score)	List of different configurations to achieve this number of successes	Calculation of probability of achieving this number of successes	Number of players (as good as Mike) who achieve this prize score	% of players who achieve this prize score
0	MMM	$\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$	8	30%
1	SMM, MSM, MMS	$\frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} \times 3 = \frac{12}{27}$	12	44%
2	SSM, SMS, MSS	$\frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} \times 3 = \frac{6}{27}$	6	22%
3	SSS	$\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$	1	4%
Total:	**8**		**27**	**100%**

Step 16

Let's compare this to the average person who shoots at 25%.

Number of successes (prize score)	% of players who achieve this prize score shooting at 25%	% of players who achieve this prize score shooting at 33.33%
0	42%	30%
1	42%	44%
2	14%	22%
3	2%	4%
Total:	100%	100%

We can see that there's only a very slight increase in players who will be paid out the 2 or 3 shot category of prize. But the vast majority of people are still only scoring on none or one of their shots.

Step 17

This will mean the game operator has to adjust the value of the prizes in each category, but only slightly.

Independently, students create another pricing table (Appendix B) and calculate the game operator's profit/loss overall.

Remind students to make the game feel fair and tempting for players, yet still ensure the game operator makes a profit.

Provisions for extending students:

Ask students to find the probability of players who score at a range of different rates (50%, 66% etc.).

How accurate would the average player have to be before this game becomes unprofitable for the game operator?

Part C:
Poker machines, skill, chance, and payout probabilities

Step 1

Reflect with students that this exercise has shown how the designer of gambling games know the mathematics behind the game and produce a game that favours the operator.

The game operator in the basketball game has determined the risks, and knows they will have to pay out a big jackpot prize to the very small percentage of people who score three out of three times. But they also know that the vast majority of people (around 80%), on average, will score none or one basket, which is where they can make their profit.

Step 2

What does this mean for other types of gambling?

Students might think they have a fair understanding of sporting matches, and that this skill and understanding increases their chances of winning. However, as this game has proven, even someone with a lot of skill is expected to lose a lot more than they win. Ask students, would betting companies really run a system where you can accurately predict the winner of a sporting match or race more than 90% of the time? How would they make profit from that?

Just like the basketball game operator, betting companies know they will have to payout, and sometimes big. But they also know how much, on average, they can expect to keep - the punters who are shooting 0/3.

Step 3

Let's take a look at poker machines.

Poker machines in NSW are legally required to payout 85% of what is gambled.

This means that for every $100 gambled on a poker machine, the machine pays back $85, and the gambling company keeps the other $15.

But this does not mean it has to pay out at a rate of 85% to **each player**.

If we think about this in terms of our basketball game (using the same success percentages as a person shooting at 33% success rate):

Prize 'score'	Number of 'winners' of this prize category	Amount gambled	Value of payout	Total cost of prizes	Profit/Loss for game operator
0	30	30 x $1 = $30	$0.10	30 x $0.10 = $3	$30 - $3 = +$27
1	44	44 x $1 = $44	$0.50	44 x $0.50 = $22	$44 - $22 = +$22
2	22	22 x $1 = $22	$1.50	22 x $1.50 = $33	$22 - $33 = -$11
3	4	4 x $1 = $4	$6.75	4 x $6.75 = $27	$4 - $27 = -$23
Total:	**100**	**$100**	-	**$85**	+$15

In this way we can see that while the majority of players don't win playing poker machines, some do, and some even win big (turning $1 into $6.75 is a pretty big win, relatively speaking.)

But overall, the poker machine pays out 85% and the gambling company still keeps a profit of 15% of everything gambled.

Why always payout a minimum of 10c? Because psychologically it keeps players feel like they're winning, meaning they'll feed more money into the poker machine, and it isn't actually costing the gambling company overall.

Step 4

There's an important difference between poker machines and the basketball game, however. There's no way to press the poker machine button in such a way that gives you an advantage. There is no way to increase the odds of a poker machine paying out through skill. The poker machine runs on probabilities and follows them robotically.

Part D: Reflection

Ask students to write a short paragraph reflecting on their approach to poker machines and games of chance, knowing the probability of winning, and the schedule on which these games are expected to pay out.

How do students feel seeing someone win a big payout on a poker machine? It has to happen, the law says a poker machine must pay out, so statistically it will, just like it's likely at least one person will shoot 3/3 basketballs and win the major prize at the Easter Show.

But for every big winner, how many people are receiving no or a very small prize?

What impact might this have on their behaviour? Will they continue to gamble on a big win, or will they realise this might not be a rewarding course of action?

Teacher reflection

Take this opportunity to reflect on your own teaching:

What did you learn about your teaching today?

What worked well?

What didn't work so well?

What would you share?

Where to next?

How are you going to get there?

Appendix A: Calculating percentages of winners - two per student

Number of successes	List of different configurations to achieve this number of successes	Calculation of probability of achieving this number of successes	Number of players who achieve this prize score	% of players who achieve this prize score

Appendix B: Calculating game operator profit/loss - two per student

Prize 'score'	Number of 'winners' of this prize category	Value of sales ($3 per game)	Value of prize	Total cost of prizes	Profit/Loss for game operator