3

Casino games – Roulette and Keno

NSW GOVERNMENT

February 2024

Activity introduction

Quick summary

Lotteries have been around for literally thousands of years. The earliest evidence we have is from China, around 200 BCE. It is even possible that those lotteries helped to finance the Great Wall of China. Even today, many governments organise their own lotteries and use the proceeds to fund projects.

In this lesson, students analyse how the operators of lottery-type games have arranged the payouts so that they are always making a profit, and that their profits accumulate by much more and much faster than any single player's winnings.

Learning intentions

Students will:

- understand that the operator of the game arranges the odds so that the operator will always win.

21st-century skills

Critical thinking

Ethical behaviour

Problem solving

Syllabus outcomes

Mathematics Standard (Year 11)

- **MS11-5** models relevant financial situations using appropriate tools
- **MS11-7** develops and carries out simple statistical processes to answer questions posed.

Mathematics Extension 1 (Year 11)

- **ME-A1** working with combinatronics.

Mathematics Life Skills (Years 11 and 12)

- **MLS-S2** probability.

Stage 6 Mathematics Syllabus Statements

Students develop awareness of the applicability of algebra in their approach to everyday life. Students analyse different financial situations, to calculate the best options for given circumstances, and solve financial problems. They develop the ability to make informed financial decisions, to be aware of the consequences of such decisions, and to manage personal financial resources effectively. Students develop an ability to justify various types of financial decisions which will affect their life now and into the future.

Knowledge of statistical analysis enables the careful interpretation of situations and raises awareness of contributing factors. Study of statistics is important in developing students' understanding of the contribution that statistical thinking makes to decision-making in society and in the professional and personal lives of individuals.

Students model theoretical or real-life situations involving algebra. Students develop knowledge, skills and understanding to manipulate, analyse and solve polynomial equations. Students use algebraic and graphical techniques to describe and solve problems and to predict outcomes with relevance to, for example, commerce.

Students develop an understanding of the language and elements of chance and probability and apply this in real situations.

Topic
Probability

Unit of work
Mathematics Stage 6

Time required
55 minutes

Level of teacher scaffolding
Medium – students will need to be heavily scaffolded through the explicit teaching of the probabilities of this trial. However, they will require minimal support when running the trial.

Resources required

- Device capable of presenting videos
- Scientific calculators – One per student
- Whiteboard

Keywords
Gambling, betting, sports, casino, money, gaming, probability.

Teacher worksheet

Teacher preparation

Gambling can be a high-risk activity and is a priority concern for young people. Therefore, before conducting the lesson on gambling, it is recommended that teachers and parents read the Facilitator pack. The pack provides teachers and parents with essential information about gambling harm amongst young people and clarifies the nature of gambling-related behaviours and how to approach sensitive topics.

Learning intentions

Students will:

- understand that the operator of the game arranges the odds so that the operator will always win.

Success criteria

Students can:

- use combinatorics to calculate the number of ways an event can occur
- calculate the fair odds for Roulette and Keno.

Teaching sequence

25 minutes - Part A: Handshakes, birthdays and counting

15 minutes - Part B: Keno

10 minutes - Part C: Roulette

5 minutes - Part D: Reflection

Part A: Handshakes, birthdays and counting

Work through this resource material in the following sequence:

Step 1

The handshake problem

Pose the following questions to students.

- **If there are 4 people meeting each other for the first time, how many handshakes are required so that everyone shakes everyone's hand exactly once?**
  3 + 2 + 1 = 6 handshakes
- **What about if there are 5 people?**
  4 + 3 + 2 + 1 = 10 handshakes
- **What if there are *n* people? Can we write a rule for this?**
  Number of handshakes = (n-1) + (n-2) + ...3 + 2 + 1.

The first person does not need to shake hands with themselves, so they have (n-1) handshakes.

The second person does not need to shake hands with themselves or the first person, so they have (n-2) handshakes. etc

Step 2

Same birthday

Pose the following questions to students.

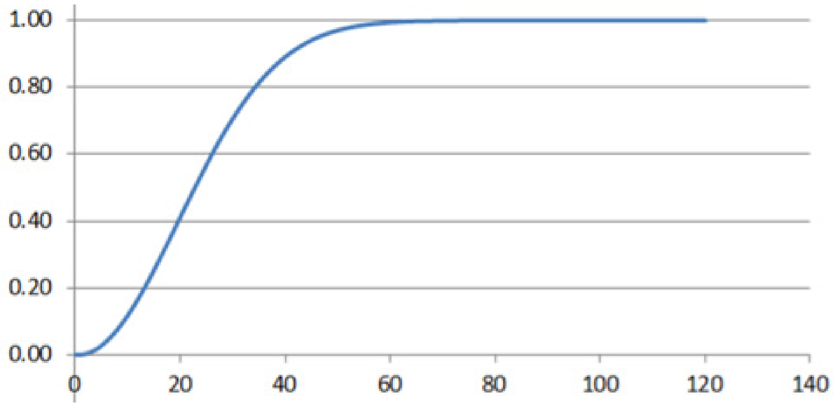
- **If 2 people meet for the first time, what is the chance of them having the same birthday?**
  Set the first person's birthday as any given date. The probability of the second having the same birthday is $1/365$.
- **Estimate the chance of 2 people in a group of 5 having the same birthday? (estimate, do not try and calculate it)**
  For 5 people the calculation is more difficult because you need to work out how many combinations of two people there are from 5 people. This calculation is done using combinatorics. But in this question we were asked for an estimate and we can accept 'not very likely'.
- **If there were 30 people in a room. Would you say it was more or less likely that 2 of them will have the same birthday?**
  Once again it seems that since we only have 30 people with birthdates out of a possible 365 different dates, it seems unlikely.

Step 3

In fact, since the number of combinations of two birthdays increases dramatically as *n* increases there is a turning point at *n*=23.

$$\Pr(n>23) > 0.5$$

The graph is shown below. By the time you get to 60 people the probability is nearly 1. The mathematics is shown later, after a bit of theory.



As we have said previously, the number of ways we can choose *r* from *n* objects of them is given by:

$$^{n}C_{r} = \frac{n!}{(n-r)! \times r!}$$

The calculations involve factorial numbers, the symbol for which is the exclamation mark: !

e.g., $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Note: Steps 4 to 6 are relevant if this lesson is being used to introduce combination notation. Otherwise, these steps can be skipped.

Step 4

Independently, students calculate

a. 6!

b. 8!

Answers:

a. $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

b. $8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40{,}320$

Step 5

We can calculate

$$\frac{8!}{6!}$$

using a shortcut as follows:

$$\frac{8!}{6!} = \frac{8 \times 7 \times 6!}{6!} = 8 \times 7 = 56$$

Step 6

Independently, students use the shortcut to calculate

a. $\dfrac{10!}{8!\ 2!}$

b. $\dfrac{200!}{199!\ 1!}$

Answers:

a. $\dfrac{10!}{8!2!} = \dfrac{10 \times 9 \times 8!}{8!2!} = 45$

b. $\dfrac{200!}{199!1!} = \dfrac{200 \times 199!}{199!1!} = 200$

Step 7

Back to the birthday problem.

It is time-consuming to calculate the probability of getting at least 1 pair of matching birthdays out of 23 people as we also need to consider the possibilities of having 3 birthdays the same, 4 birthdays, 5 birthdays, all the way up to all 23 people having their birthday on the same date.

It is easier instead to calculate the possibility of them all being different and subtract from 1.

This is called using the complement of the event we are investigating.

Step 8

How many pairs could we potentially make from 23 people? That is, how many different ways can we choose 2 people from 23?

$$^{23}C_2 = \frac{23!}{(21)! \times 2!} = 253 \text{ different pairs, each of which must be different.}$$

Step 9

Every pair must contain 2 individuals with a different birthday otherwise there will be a match in the whole group of 23. The probability that 2 people in a pair have a different birthday is $364/365$ (as the second person must have a different birthday to the first).

Since we need all 253 pairs to have different birthdays the probability is:

$$\left(\frac{364}{365}\right)^{253} \approx 0.4995$$

The probability of at least one matching pair becomes 1 - 0.4995 = 0.5001.

That means that with 23 people there is roughly 50% chance that 2 people will share the same birthday.

Part B: Keno

Step 1

Explain to students that now that we know how to count combinations, we can look closely at the game of Keno.

This is very popular in clubs across Australia and increasingly online.

It is a simple game with the added attraction of seeming easy to win. In clubs and online a new game begins every 3 or 4 minutes and the results are known immediately. (If you win more than $2,000 you may need to wait until the next day to collect your prize.) The advertised maximum prize for each draw is $200,000 for choosing 10 numbers correctly.

Step 2

Explain the rules of Keno to students:

- A player selects 10 numbers from between 1 and 80.
- 20 numbers are drawn, with no number being repeated.
- If all the 10 numbers you have selected are among the 20 then you win the top prize.
- Smaller prizes are also awarded for getting some of the numbers correct.
- You don't have to pick 10 numbers and instead can choose fewer numbers -if you decide to bet on one number only it is called a Draw 1. If you decide to bet on 2 numbers, it is called a Draw 2 etc.
- **Important:** Make sure students understand this point. If you correctly guess 5 numbers but bet on a Draw 7, you still receive a payout, though a smaller one.
- Prizes are pre-calculated using:

$$\frac{\textit{number of winning combinations}}{\textit{total number of combinations}}$$

- Prizes are expressed as a return for each $1 bet.

Step 3

Ask students to imagine a $1 Draw 1 bet is placed on the number 5.

Ask students to find the chance that 5 is one of the 20 numbers drawn? There are 20 numbers drawn out of 80 possible numbers, so the probability any single number is in the winning draw of 20 is:

Answer:

$$\frac{20}{80} = \frac{1}{4}$$

Step 4

The payout for each $1 in a Draw 1 game is $2. As a class, discuss whether students think this is fair?

For the game to be fair, the payout should equal the reciprocal of the probability that you win. So if the chance of winning is ¼ the fair prize would be $4. This is not fair and guarantees the game operator success in the long run.

Step 5

If the payout for a Draw 1 was $2, ask students to estimate the payout for a Draw 2. Will it be more or less?

Ask some students to volunteer their reasoning.

Two successful numbers are less likely than 1 successful number, so you would rightly expect the payout to be greater.

However, it is a common mistake for students to estimate the payout should be $4. They think 2 numbers is twice as hard as 1 number.

Step 6

Explain to students that in fact the payout for Draw 2 is $10. Does this seem reasonable? What are the actual odds of selecting two numbers correctly?

Explain to students that the odds of successfully selecting two numbers are actually around 1 in 17. There are 3 alternate ways that the chance of success in a Draw 2 can be calculated:

- $\frac{2}{80} \times \frac{1}{79} \times {}^{20}C_2 = \frac{19}{316}$ where $\frac{2}{80}$ is the probability that the first number drawn is a match, $\frac{1}{79}$ is the probability that the next number drawn is a match and given 20 numbers are drawn there are ${}^{20}C_2$ ways that the matching 2 numbers can be found in the 20 numbers actually drawn.
- $\frac{20}{80} \times \frac{19}{79} = \frac{19}{316}$ where $\frac{20}{80}$ is the chance that your first number is contained in the 20 numbers picked and $\frac{19}{79}$ is the chance that your other number is contained in the remaining 20 numbers picked.
- ${}^{20}C_2 \div {}^{80}C_2 = \frac{19}{316}$ where ${}^{20}C_2$ is the number of ways your 2 numbers can be picked within the 20 drawn and ${}^{80}C_2$ is the total number of ways your 2 numbers can be picked from the total of 80 numbers. Note that most calculators cannot compute combinations of this magnitude. However, the function COMBIN(n,r) in Excel can be used to evaluate these large combinations.

Does this change their impression of what the payout should be?

Based on an actual probability of a successful Draw 2 of $\frac{19}{316}$ the actual payout should be:

$$\frac{316}{19} \approx \$16.63 \text{ or } \$17 \text{ (to the nearest dollar).}$$

Step 7

Show students this list of approximate odds for success on Keno draws.

Type of bet	Actual probability of success	Fair payout (per $1 gambled) given these odds	Actual payout
Draw 1	1:4	$4	$2
Draw 2	1:17	$17	$10
Draw 3	1:73	$73	$25
Draw 4	1:327	$327	$50
Draw 5	1:1,551	$1,551	$500
Draw 6	1:7,753	$7,753	$1,500
Draw 7	1:40,980	$40,980	$5,000
Draw 8	1:230,115	$230,115	$15,000
Draw 9	1:1,380,688	$1,380,688	$25,000
Draw 10	1:8,911,712	$8,911,712	$200,000

What do students notice about this list?

Students will quickly realise that something isn't adding up here.

As the size of draw (how many numbers you need to match) increases, the chance of success rapidly diminish and the fair payout grows significantly. But the actual payout is not growing anywhere near as rapidly.

Step 8

Ask students:

- **Why is the actual payout value less than the odds?**
- **Why do you think the operator offers smaller prizes even if you do not correctly select all numbers?**
  To encourage people to keep playing by winning smaller prizes occasionally. The same reason we have different divisions in lotto games.

Step 9

Show students this list of all the odds for Draw 5 and Draw 10. Note the smaller prizes for correctly selecting fewer winning numbers.

Why do students think the allocated prize decreases for fewer correct numbers? Encourage students to think beyond it is harder to get more numbers correct.

Draw 5	Number correct	Odds of winning	Prize
5	5	1:1,551	$500
5	4	1:83	$15
5	3	1:12	$2

Draw 10	Number correct	Odds of winning	Prize
10	10	1:8,911,712	$200,000
10	9	1:163,382	$10,000
10	8	1:7,385	$500
10	7	1:621	$50
10	6	1:88	$10
10	5	1:20	$3
10	0	1:22	$3

Answer: As fewer numbers are easier there will be an increase in the number of people winning.

Take the 4 out of 5 and 3 out of 5 as an example.

$$83 \div 12 \approx 7$$

There will be 7 times the number of winners for 3 as for 4. This means the Keno operator has to payout more winners, so they can't offer as high prizes, because they can't afford them.

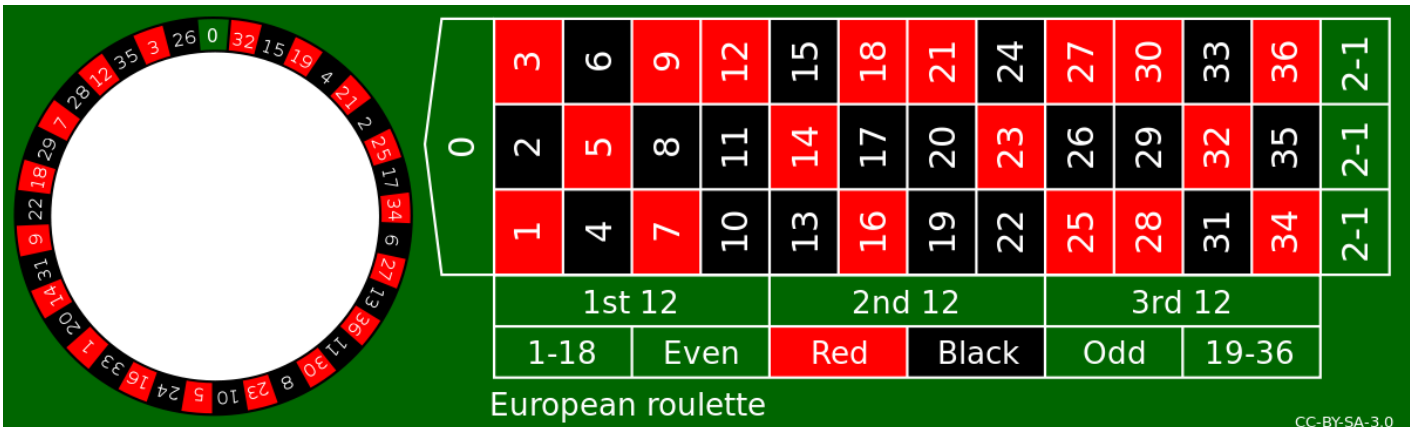
You can also see that $2 \times 7 = 14$ the payout for correctly selecting 4 numbers.

Part C: Roulette

Step 1

Understanding roulette

Roulette is a classic casino game played by betting on the finishing position of the ball when the wheel is spun. The numbers 1 to 36 are on the wheel, 18 of which are red and 18 black. We will ignore the possibility of 0 (or 00 or 000 in the US) for the moment.



Bets are placed on the green mat using plastic chips purchased from the casino.

The odds for each bet can be calculated according to the probability of occurring.

Step 2

Calculating the chance for each bet.

The basic probability of any event is found using the formula:

$$Pr(\text{an event}) = \frac{\text{number of ways the event can happen}}{\text{total number of outcomes}}$$

We will come back to the green 0 later but for the moment let's calculate using just the 36 red and black numbers.

For the 3 options: odd or even, 1 to 18 or 19 to 36, and red or black this equation shows:

$$Pr(\text{each event}) = \frac{18}{36} = \frac{1}{2}$$

The odds provided by the casino are 2:1. If you bet $1 you win $2, $1 profit. If you bet $100 you win $200 etc. This is a fair game. Both parties are investing the same amount on the outcome of a 50:50 event.

Step 3

Independently, students use the formula to calculate the odds of winning.

a. 1st 12 (i.e. a number from 1 to 12)

b. The first column (i.e. numbers 1,4,7,10...)

c. Placing a bet on the number 8.

Answers:

a. $Pr(1st\ 12) = \frac{12}{36} = \frac{1}{3}$ = the casino pays \$3 for a \$1 bet so again "fair."

b. $Pr(1st\ column) = \frac{12}{36} = \frac{1}{3}$ = the casino pays \$3 for a \$1 bet so again "fair."

c. $Pr(8) = \frac{1}{36}$ = the casino pays \$36 for a \$1 bet so again "fair."
You may also bet on a group of 6 numbers ($Pr = \frac{1}{6}$)
or other possibilities depending on the casino.

The operator pays exactly the odds that you would expect in a fair game so how do they make a profit? This is where the green 0 comes in. When the green 0 appears, **all bets lose.** This is what gives the house their advantage. In the long run they win 1 in 37 spins. In the US wheels also have a double 00 which increases the house chance to winning 1 in 18 spins and a much bigger profit.

Part D: Reflection

Summarise what we have learned through this lesson.

- Probability and combinatorics allow us to calculate the theoretical probability of success in these games.
- The designers of the games use the same calculations to make sure that the odds are in favour of the operator.

Questions to ponder:

- **Why is Keno popular with gamblers and how do the operators make a profit?**
  Keno is popular because correctly choosing a few numbers correctly out of 20 drawn seems much easier than it is. The results are known almost immediately. The operators know the chances of winning and in the long run will take about 30% for profit and taxes.
- **Why do casino operators supply perks like free accommodation, meals and drinks for high rollers (people who bet large amounts)?**
  They know that there is a fixed percentage that they will win from these gamblers. The more they gamble the more the casino wins.
- **On first look it seems that Roulette only wins 1 in 37 spins which is only 2.7% of the money invested. They seem to be taking much less than the operators of games like Keno and lotto. The operator makes a much bigger percentage than this. How do they do it?**
  There is a cumulative effect because most people reinvest any winnings they get. In the long run the gambler loses 2.7% of their initial pot of money, then another 2.7% of their winnings, then another 2.7% of their winnings etc until they give up and walk away with whatever money they have left.

Teacher reflection

Take this opportunity to reflect on your own teaching:

What did you learn about your teaching today?

What worked well?

What didn't work so well?

What would you share?

Where to next?

How are you going to get there?