4

Lotto mania

February 2024

Activity introduction

Quick summary

Being struck by lightning. Being bitten by a shark.
Which is more likely?

Lotteries are popular because of their accessibility and low stakes, and their huge possible jackpot payout, which could be life-changing for many gamblers. It can be a fun pastime, and generate many positive feelings as you sit and watch hopeful that your numbers come up.

So, what are the chances of winning a Division 1 payout? And are there any viable strategies for increasing our chances?

In this lesson, students will simulate a lottery, and perform combinatorics calculations to determine the small probability of their exact numbers being drawn from a lottery. They will also consider the financial investment that would be required to guarantee a lottery win, ultimately determining that such a strategy would end up costing far more than you might win.

Learning intentions

Students will:

- understand the allocation of prize money in lottery-type games
- understand the operator of the game always wins by taking out a set percentage from the money gambled.

21st-century skills

Critical thinking

Digital literacy

Ethical behaviour

Problem solving

Syllabus outcomes

Mathematics Standard (Year 11)

- **MS11-5** models relevant financial situations using appropriate tools
- **MS11-7** develops and carries out simple statistical processes to answer questions posed.

Mathematics Life Skills (Years 11 and 12)

- **MLS-S2** probability.

Stage 6 Mathematics Syllabus Statements

Students develop awareness of the applicability of algebra in their approach to everyday life. Students analyse different financial situations, to calculate the best options for given circumstances, and solve financial problems. They develop the ability to make informed financial decisions, to be aware of the consequences of such decisions, and to manage personal financial resources effectively. Students develop an ability to justify various types of financial decisions which will affect their life now and into the future.

Knowledge of statistical analysis enables the careful interpretation of situations and raises awareness of contributing factors. Study of statistics is important in developing students' understanding of the contribution that statistical thinking makes to decision-making in society and in the professional and personal lives of individuals.

Students develop an understanding of the language and elements of chance and probability and apply this in real situations.

Topic
Probability

Unit of work
Mathematics Stage 6

Time required
60 minutes

Level of teacher scaffolding
High – students will require significant scaffolding to work through the possible combinations and to understand how the model is operating.

Resources required

- Appendix A – Question sheet
- Appendix B – Lottery model tracker
- Calculators – one per student
- Die – 2 six-sided
- Student workbooks

Keywords
Gambling, betting, sports, casino, money, wellbeing, gaming, probability, lottery, lottery, jackpot, combination, factorial notation.

Teacher worksheet

Teacher preparation

Gambling can be a high-risk activity and is a priority concern for young people. Therefore, before conducting the lesson on gambling, it is recommended that teachers and parents read the Facilitator pack. The pack provides teachers and parents with essential information about gambling harm amongst young people and clarifies the nature of gambling-related behaviours and how to approach sensitive topics.

Learning intentions

Students will:

- understand the allocation of prize money in lottery type games
- understand the operator of the game always wins by taking out a set percentage from the money gambled.

Success criteria

Students can:

- run a simulation of a lottery type game
- calculate prizes for lottery type games given the prize pool and the number of winning entries
- demonstrate how winnings are not related to the numbers that are drawn, only the number of entries
- demonstrate how lottery operators profit from this system.

Teaching sequence

10 minutes - Part A: What is the lottery?

20 minutes - Part B: Sharp Steve's chances of winning

20 minutes - Part C: Modelling a lottery - the house always wins

10 minutes - Part D: Reflection

Part A:
What is the lottery?

Work through this resource material in the following sequence:

Step 1

As a class, discuss the idea of a national lottery. Prompt student thinking with the following questions:

- What examples have you seen of lotteries? Where?
- Do you understand how a lottery works? Can you explain it?
- Why do people enter these gambling competitions?
- Do you think the people entering these games really understand what their chances of winning are?

Often people enjoy engaging with lotteries due to the low cost of entry, the low amount of skill involved, compared to a potentially massive, multi-million dollar jackpot payout, which they believe they have a reasonable chance of winning.

Step 2

Explain to students that for this lesson you are going to design and run a lottery in order to calculate the odds of winning.

This lottery will follow these rules:

- There are 45 numbers in the pool.
- Players choose six numbers.
- Each entry costs $0.70 (70c to guess one set of six numbers).
- Six numbers are drawn from the pool at random.
- A number is not placed back into the pool once it is drawn (so no number can be drawn twice).
- A number is not replaced by a different number once it is drawn. **Why is this important?** Because the odds of choosing a specific number on the fifth 'draw' will be different compared to the first 'draw'.
- If you get all six numbers correct, you win the Division 1 prize.
- The amount of numbers required to be guessed correctly to win decreases with each division.

Division	Amount of numbers required to be guessed correctly to win (/6)
1	6
2	5
3	4
4	3
5	2

- If more than one person wins a division, the prize pool is divided by the number of winners.

Step 3

Ask students to estimate their chance of winning Division 1 with a single entry. That is, if they only guess six numbers, what is the probability of all six of those numbers being drawn out of the pool of 45?

A common response is an estimate of 1 in a million. This is good and shows that students think it is unlikely. Ask them why is it so unlikely?

Part B: Sharp Steve's chances of winning

Step 1

Explain to students that in order to win, we need to guess the first number correctly, and the second number correctly, and the third number correctly, and the fourth number correctly and the fifth number correctly and the sixth number correctly.

Remember 'and' means multiply in probability.

Since the first 'draw' could be any of the six numbers we guessed and we would still be on a path to winning, the first draw has a probability of $\frac{6}{45}$ of being drawn correctly in our favour.

Remember that the number of balls left in the total pool reduces by 1 each time, as does the number of balls remaining that could be a correct guess for a player.

So the equation would be:

$$\frac{6}{45} \times \frac{5}{44} \times \frac{4}{43} \times \frac{3}{42} \times \frac{2}{41} \times \frac{1}{40}$$

Step 2

Have students independently calculate the probability of winning the Division 1 prize.

The answer is:

$$\frac{720}{5{,}864{,}443{,}200}$$

Simplified down this is:

$$\frac{1}{8{,}145{,}060}$$

This is about 1 in 8.1 million chance of winning Division 1.

Note: This calculation can also be done using combinatorics.

Part B: Sharp Steve's chances of winning

If we have *n* objects the number of ways we can choose *r* of them is given by

$$^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

The calculations involve factorial numbers, the symbol for which is !

For example, $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

In this instance we wish to calculate how many different combinations there are when we choose 6 from 45.

$$^{45}C_{6} = 8{,}145{,}060$$

Step 3

Of course, people rarely just put in one entry to the lottery. In practice, people place multiple entries at the same time, with each different group of numbers costing $0.70.

Propose to students that Sharp Steve decides he can ensure winning by covering all possible combinations of 6 numbers.

The following questions can also be found in Appendix A. Answers have been provided below for your convenience.

- **How many entries would he need to make?**
  8,145,060 entries to cover every possible outcome.

- **At $0.70 an entry, how much would this cost?**
  $5.7 million.

- **Is Sharp Steve's idea of covering all possible outcomes a good idea? Will he win money?**
  Most jackpots are between $100,000 and $1 million. And this is before the division is split between multiple winners. Sharp Steve's strategy could technically work if the lottery paid out more than $5.7 million and he was guaranteed to be the only winner.

- **Sharp Steve decides that the numbers 1, 2, 3, 4, 5 and 6 are incredibly unlikely to be drawn, so he excludes this combination. Is that a good strategy?**
  Every different group of numbers has the same chance of appearing. 1, 2, 3, 4, 5, 6 is no less likely than any other set of numbers. Not covering that combination will save him 70 cents, but may cost him the first prize.

- **Let's help Sharp Steve get a feel for how unlikely winning a lottery game is. If he entered once per week, how many years would it take him, on average, to win once?**
  Steve would have to put in ~8.1 million entries, which is ~8.1 million weeks. Dividing by 52 weeks in a year, this is 156,636 (rounded up) years.

- **Sharp Steve doesn't think he'll last that long. He reckons he's only got another 50 years left, and he'd like to win Division 1 at least once in that time. How many entries would he have to put in each week for the next 50 years (to get to ~8.1 million entries in that time)?**
  50 × 52 = 2,600 weeks. 8,145,060 ÷ 2,600 = 3,133 entries per week.
- **How much would this cost him (per week, at $0.70 per entry)?**
  Sharp Steve would have to spend $2,193 on lottery entries *every week* for *50 years* to have this chance at winning Division 1.

Step 4

Can students see any problems with Sharp Steve's outrageous strategy?

Prompt their thinking by asking, just because Steve has already put in one entry of a combination of numbers, does that reduce the total amount of combinations that could be drawn?

The lottery company doesn't care what Steve has already entered. He could put in a combination of 1, 2, 3, 4, 5, 6, and 25 years later, that exact combination could be drawn. But Steve has already crossed it off his list!

In reality, Steve would have to enter 8.1 million combinations every single week/draw of the lottery to ensure he wins.

Step 5

Explain to students that the assumption made is that every one of the 8,145,060 combinations will be drawn before any single combination repeats. This is of course nonsense as it would require the balls to have a memory, so they know whose 'turn' it is to be drawn.

People incorrectly think that just because a random event has not occurred for a while the probability of it occurring has increased.

For example, if you are tossing a single coin and toss 4 heads in a row, some people might believe the probability of tossing a tail has increased for the next toss as it is 'due'. In fact the probability of a head on the next throw remains ½, previous results do not affect future outcomes.

There is a cottage industry that revolves around people believing in the gambler's fallacy's impact on lottery numbers. Records of the frequency of the lottery numbers that are drawn weekly are kept and then this meaningless information is sold to gamblers. Armed with this information they select numbers which have not appeared for a while because they believe they are more likely to come out soon, or because they seem to have been drawn more often across the history of the lottery.

Part C:
Modelling a lottery - the house always wins

Step 1

Explain to students that, just like poker machines and race or sports betting companies, the operators of the lottery are confident that they will always keep a certain share of the money gambled, whilst expecting to pay out some of it.

They do this by tracking the probabilities of players selecting a winning combination of numbers.

Step 2

To see this in action, the class is going to model a lottery.

The rules for this lottery will be different to a 45 number draw.

- Each round costs $1 entry to play. Each player starts with $10.
- Instead of drawing numbers, we will roll a pair of six-sided dice.
- Players select a pair of numbers they hope to be rolled.
- Division 1 will pay out 30% of the pool, split evenly between winners, if a person guesses both numbers correctly.
- Division 2 will pay out 40% of the pool, split evenly between winners, if a person guesses both numbers correctly.
- The remaining 30% of the money goes to paying government taxes and a profit for the operator.
- If Division 1 is not won, the money jackpots into the next week.

Step 3

Independently, using their workbooks, students list all the possible combinations that could be rolled by a pair of dice.

Step 4

Next, students list the probability of each of these combinations appearing.

Step 5

Ask students,

- Do all combinations have the same probability of occurring?
- Is there a combination that occurs more than once?
- Is 3,4 different from 4,3?

In this case, no. It doesn't matter in what order the numbers are drawn, so long as a player guessed those numbers would appear.

This would be the same as, in a regular lottery, 1, 2, 3, 4, 5, 6 winning just as much as 6, 4, 2, 3, 1, 5, or any other combination. The player has still guessed all six numbers correctly.

So what does that mean for your lottery model?

Give students a chance to determine that rolling a double (1,1) is only half as likely to occur as rolling two different numbers because it can only be rolled in one way (3, 4 and 4, 3 would both be a win, effectively doubling the chance of this win occurring).

Then agree as a class on another rule:

- If a double is rolled, the dice will be re-rolled.

Step 6

- **What is the probability of winning Division 1 in this lottery?**
  With any combination now having two ways of being achieved, and 30 different combinations being rolled, each player has a 1/15 chance of winning.

Step 7

Give each student a copy of Appendix B: Lottery model tracker.

Step 8

Run the simulation by following these steps:

- Students select a combination by ticking it in their tracker.
- Students subtract $1 from the previous total to pay for their entry.
- Teacher rolls the dice. If it is a pair, reroll one of them.
- Students shade the winning combination on the tracker for that round so we can later look back and see how many times each pair appeared.
- Calculate the Division 1 prize using:

$$\frac{0.3 \times \$1 \times \textit{number of students}}{\textit{number of winners}}$$

  (Note the 0.3 represents the 30% of the prize pool).
  This is how much money all of the winners can add to their total.

- Calculate the Division 2 prize using:

$$\frac{0.4 \times \$1 \times \textit{number of students}}{\textit{number of winners}}$$

(Note the 0.4 represents the 40% of the prize pool).
This is how much money all of the winners can add to their total.

- If there are no winners, the unallocated prize money (30% or 40% of the total gambled) is added to the next round's prize pool. This is called a jackpot.
- Calculate the operator profit by using:

$$\frac{0.3 \times \$1 \times \textit{number of students}}{1}$$

Add this to the operator profit column on the tracker.

Step 9

As a class, discuss the following questions:

- **Who made a profit?**
- **Did anyone have a strategy for how they chose their combination each round?**
  A positive strategy is trying to avoid popular pairs to maximise the dividend if they win. A negative strategy is not choosing pairs that have already been successful. This is an example of the gambler's fallacy. If you hear this then remind students the dice do not have memories so they do not know what pairs have already been successful.
- **How could we check that 70% of the money gambled was returned as prizemoney?**
  Add each player's total. It should be 0.7 × 10 × (the number of students).
- **Who was the biggest winner from this lottery game?**
  The operator and the government who collected 30% each week.

Part D: Reflection

Summarise what has been covered throughout this lesson.

- Lottery games take a fixed percentage of the total invested.
- The operator makes this profit regardless of which numbers are drawn.
- Their profit is dependent only upon the number of entries.
- The chances of winning Division 1 are only 1 in 8.1 million.

As a class, reflect on the following questions:

- **Why do lotteries all have lots of smaller prizes and not just the Division 1 jackpot?**
  Small rewards keep people interested. If they never won they would stop playing.
- **The game designers keep a small percentage of the pool out and every now and then put it back in for a SuperDraw. Why do you think that is?**
  To gain publicity, increasing the number of people playing, and increasing their profits.

Ask students to write a short paragraph reflecting on their approach to the lottery and games of chance, knowing the probability of winning, and the schedule on which these games are expected to pay out.

How do students feel seeing someone win a big payout on the lottery? Statistically it has to happen.

But for every big winner, how many people are receiving no or a very small prize?

What impact might this have on student behaviour? Will they continue to gamble on a big win, or will they realise this might not be a rewarding course of action?

Teacher reflection

Take this opportunity to reflect on your own teaching:

What did you learn about your teaching today?

What worked well?

What didn't work so well?

What would you share?

Where to next?

How are you going to get there?

Appendix A: Question sheet

If there are 45 numbers in a lottery draw, and you have to correctly guess all six of the six numbers drawn to win, calculate the probability of winning the Division 1 prize.

Sharp Steve decides he can ensure winning by covering all possible combinations of 6 numbers.

How many entries would he need to make?

At $0.70 an entry, how much would this cost?

Is Sharp Steve's idea of covering all possible outcomes a good idea? Will he win money?

Sharp Steve decides that the numbers 1, 2, 3, 4, 5 and 6 are incredibly unlikely to be drawn, so he excludes this combination. Is that a good strategy?

Let's help Sharp Steve get a feel for how unlikely winning a lottery game is. If he entered once per week, how many years would it take him, on average, to win once?

Sharp Steve doesn't think he'll last that long. He reckons he's only got another 50 years left, and he'd like to win Division 1 at least once in that time. How many entries would he have to put in each week for the next 50 years (to get to 8.1 million entries in that time)?

How much would this cost him (per week, at $0.70 per entry)?

Appendix B: Lottery model tracker

																My total	Operator profit
Round	1, 2	1, 3	1, 4	1, 5	1, 6	2, 3	2, 4	2, 5	2, 6	3, 4	3, 5	3, 6	4, 5	4, 6	5, 6	$10	$0
1																	
2																	
3																	
4																	
5																	
6																	
7																	
8																	
9																	
10																	